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vFFA and novel optics for muon collider accelerator complex

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FFA20 workshop

Contents

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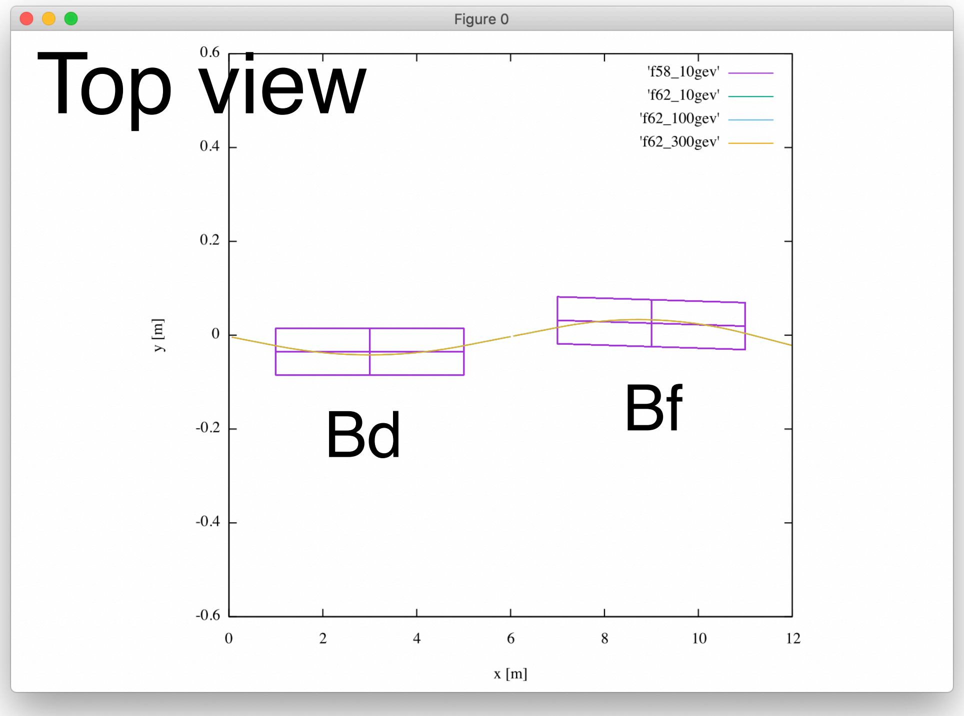
Vertical excursion FFA (vFFA)

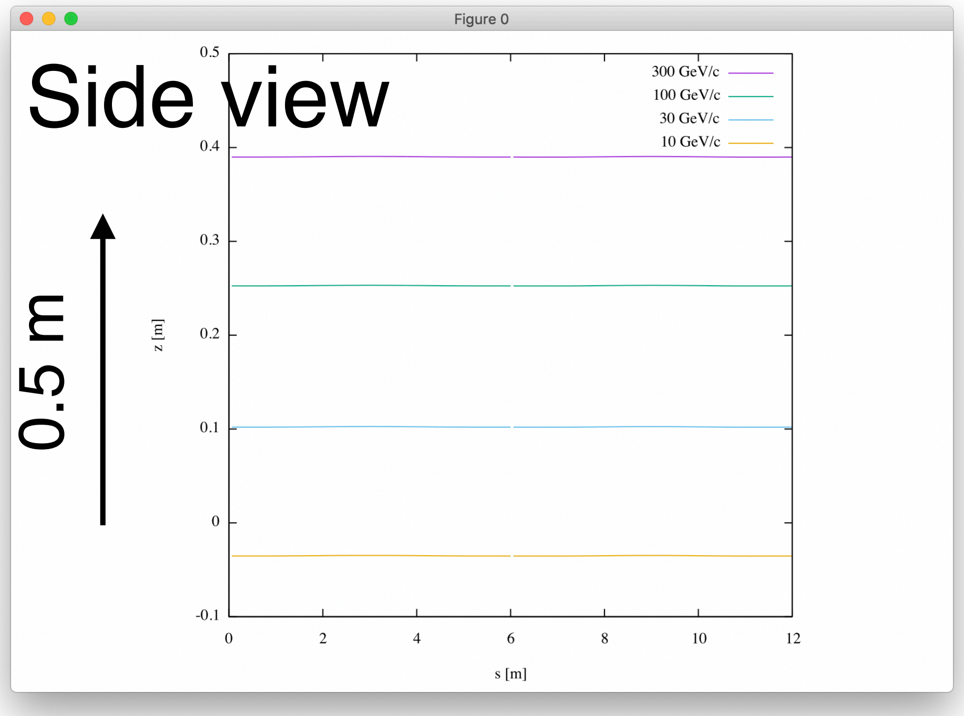
- Invented in 1955 by Tihiro Ohkawa.
- Re-invented in 2013 by Stephen Brooks.
- Orbit moves vertically when the beams are accelerated.
- **Path length is constant for all the momenta. Momentum compaction factor is zero.**
- It was called electron cyclotron.
  - Ultra-relativistic particles can be accelerated continuously with fixed field magnets.

- Ideal for muon acceleration.
  - No ramping of magnetic fields.
  - No RF frequency modulation.
  - Large momentum ratio from injection to extraction, e.g. ~30.
  - Wiggling orbits to spread out neutrino.
  - Enough vertical aperture is needed.

G8. **FFAG Electron Cyclotron.*** Tihiro Ohkawa, *University of Illinois*† (introduced by D. W. Kerst).—New types of FFAG[1] accelerators having the same orbit length for all momenta are proposed. In these types electrons, injected with an energy of a few Mev, are accelerated by a fixed frequency electric field until the radiation loss becomes serious, probably

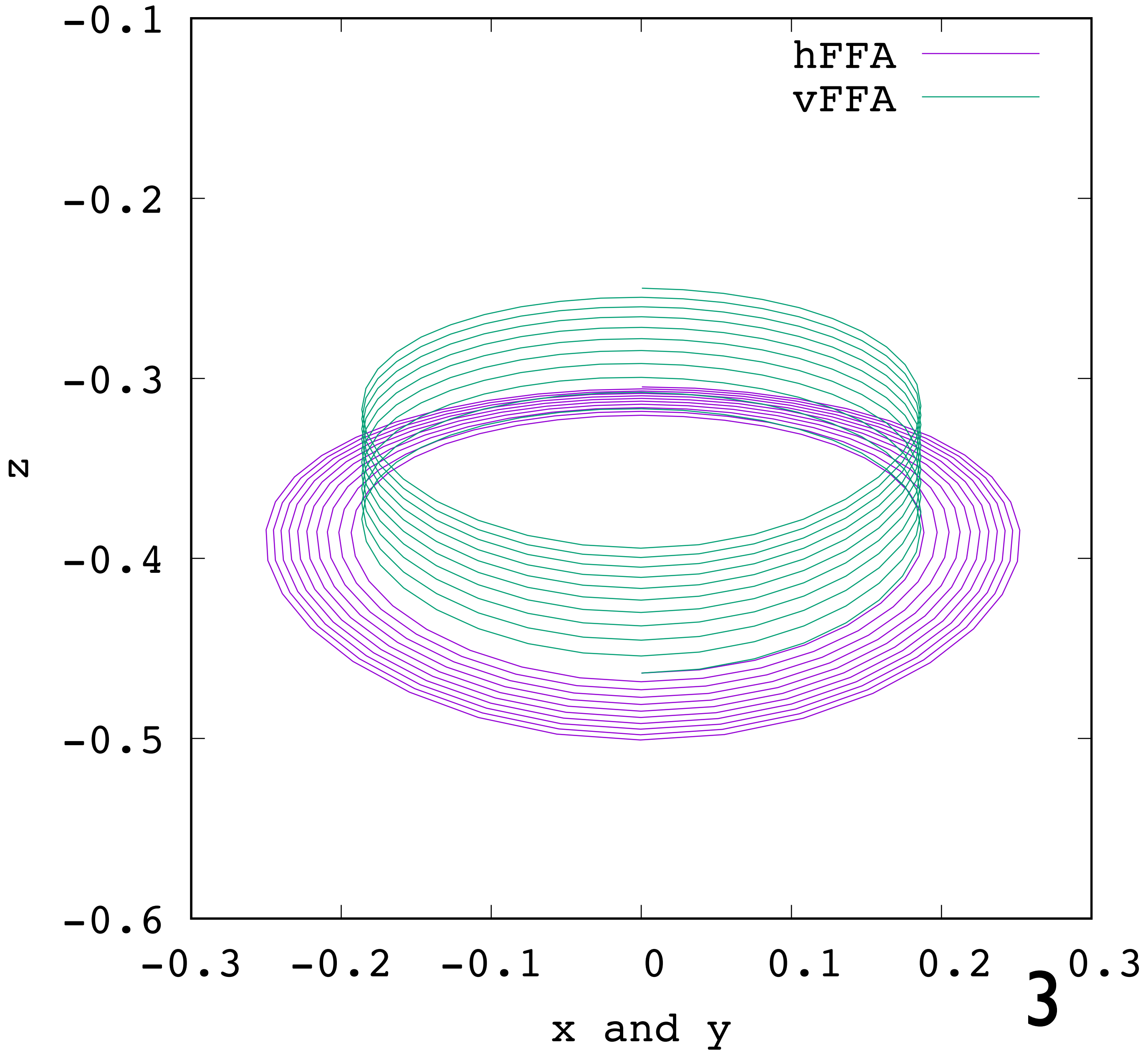
Bull. APS 30, 20 (1955)
by Tihiro Ohkawa





$$B = B_0 \exp(my)$$

m : field index
y : vertical



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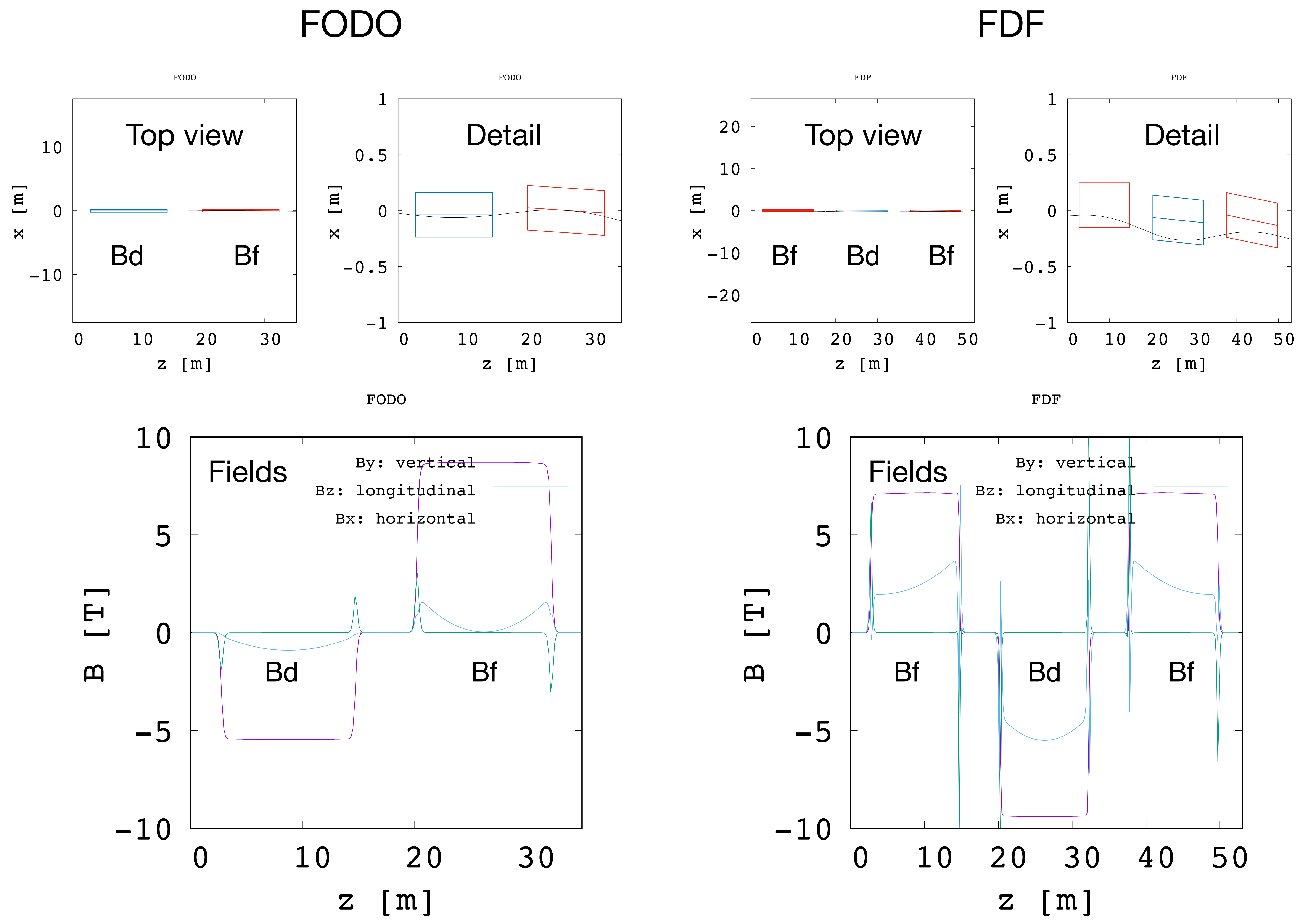
Muon accelerator ring

Design constraints

- Circumference is about LHC.
- Top energy is 1.5 TeV per beam.
- Momentum ratio from injection to top energy is about 30.
- Maximum field is no more than 10 T.
- Orbit excursion is less than 0.5 m.

1.5 TeV accelerator in LHC tunnel

	FODO	FDF
Energy	50 GeV to 1.5 TeV	50 GeV to 1.5 TeV
Cell length	35 m	52.5 m
Magnet length	2 x 15 m	3 x 15 m
# of cell	810	540
Maximum field	8.7 T	10.6 T
Field index m	6.8	3.0
Orbit excursion	0.50 m	1.13 m
Cell tune	0.3957 / 0.0861	0.3510 / 0.1515



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- Reduction of reverse bending is one of optimisation targets.

Muon collider ring (arc)

Design constraints

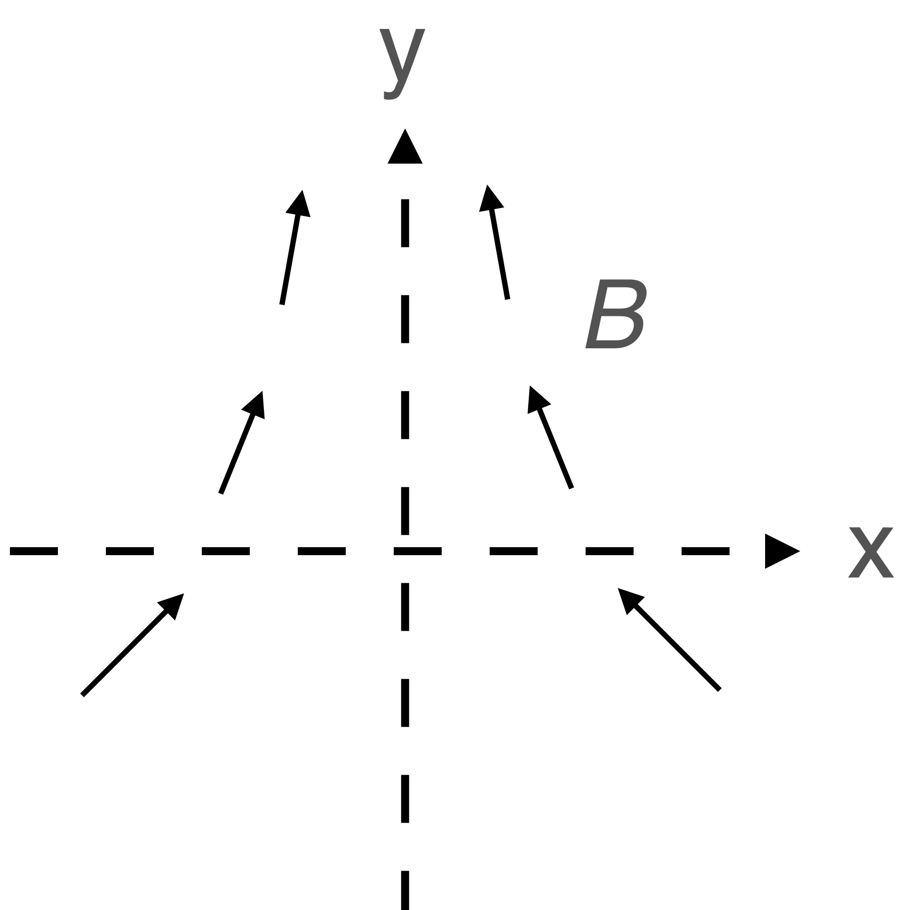
- Energy is 1.5 TeV per beam.
- Circumference should be as small as possible.
- Minimise momentum compaction factor (alpha).
- Maximum field is no more than 14 T (use the same criterion of LEMMA accumulator ring).
- Moderate momentum acceptance.

Does vFFA concept help?

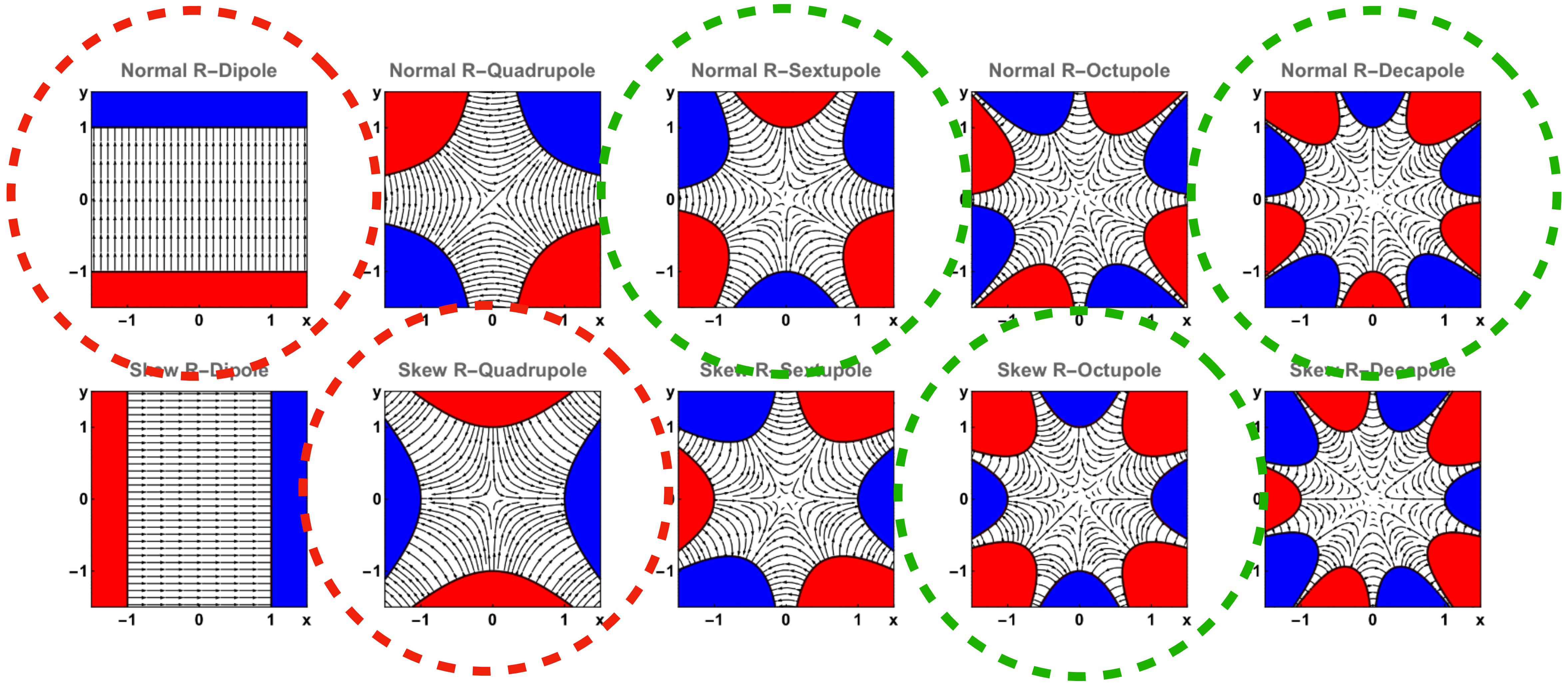
- Simplified magnet only up to skew quadrupole.
- Lattice keeping the first few multipoles is another direction (will not talk).

$$\frac{B_y}{B_0} = \exp(my)\cos(mx) \approx 1 + my + \frac{m^2}{2!}\left(y^2 - x^2\right) + \frac{m^3}{3!}\left(y^3 - 3x^2y\right) + \ldots$$

Normal Dipole **Skew** Quad **Normal** Sext **Skew** Oct



$$B = B_0 \exp(my)$$



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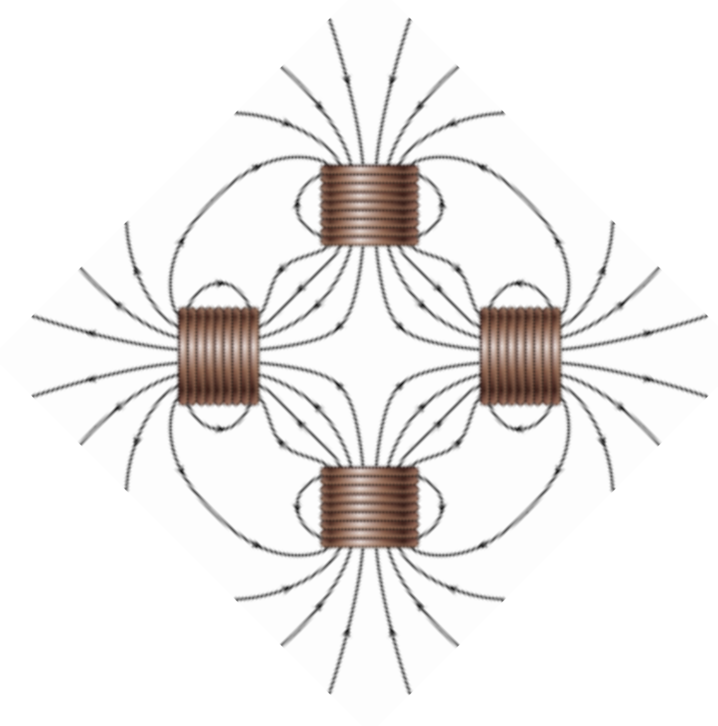
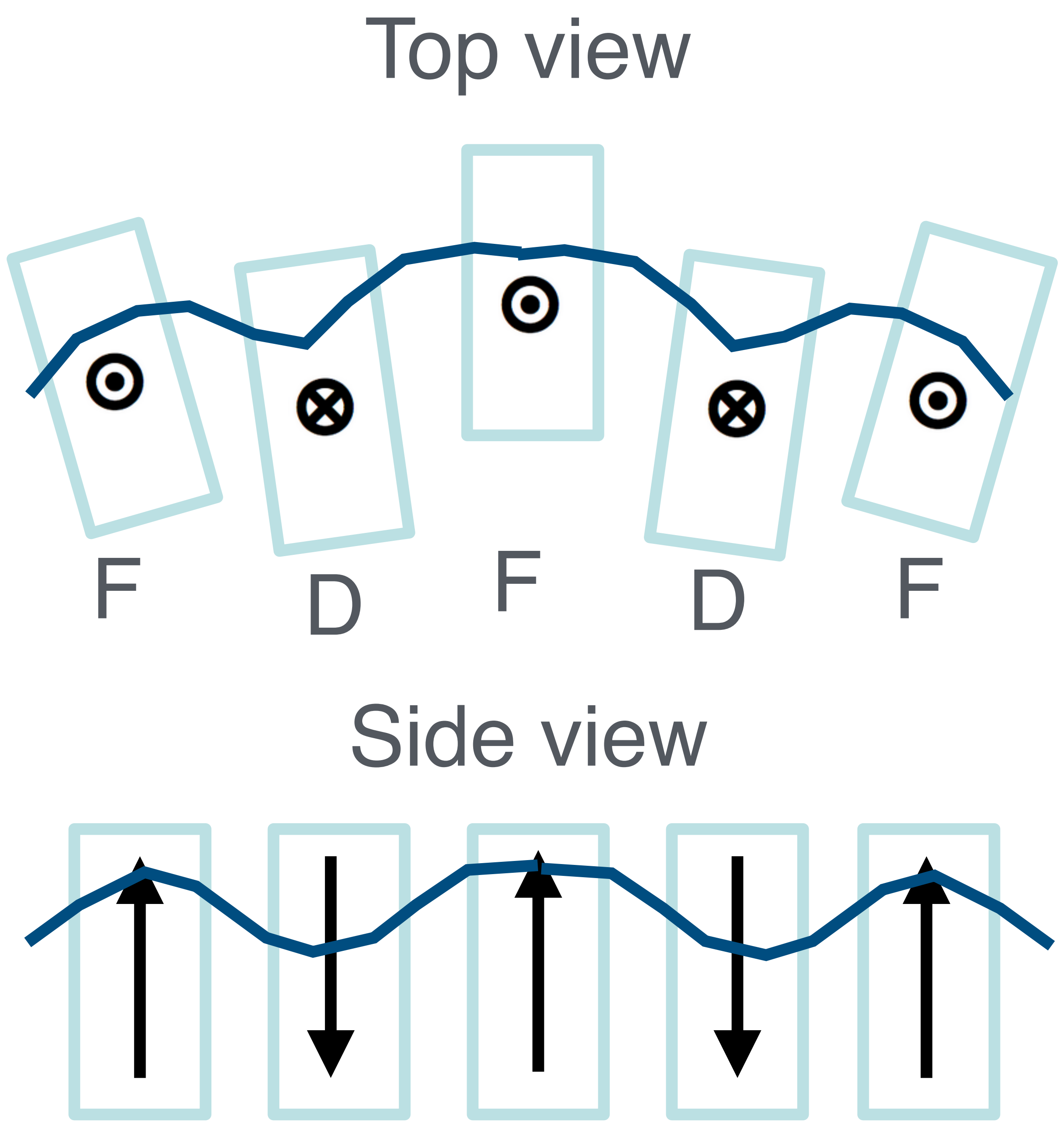
vFFA like lattice with only skew Q



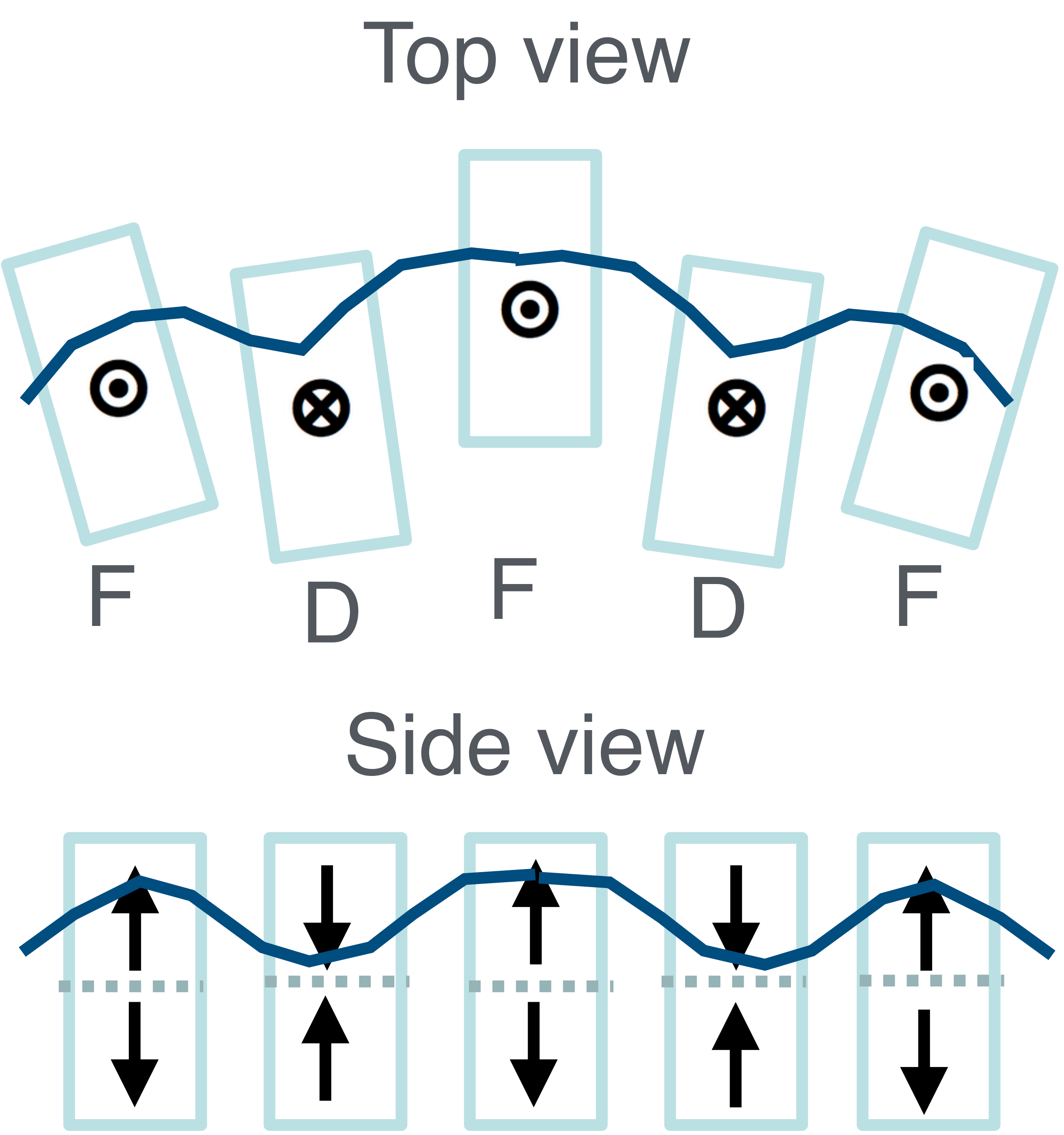
Skew quadrupole

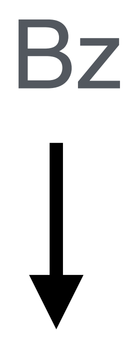
- vFFA like lattice can be made using upper half of skew Q.
- Tune depends on momentum: non-scaling.
- Momentum compaction factor (alpha) is **no longer zero**.

Scaling vFFA (FODO)



Only with skew Q (FODO)





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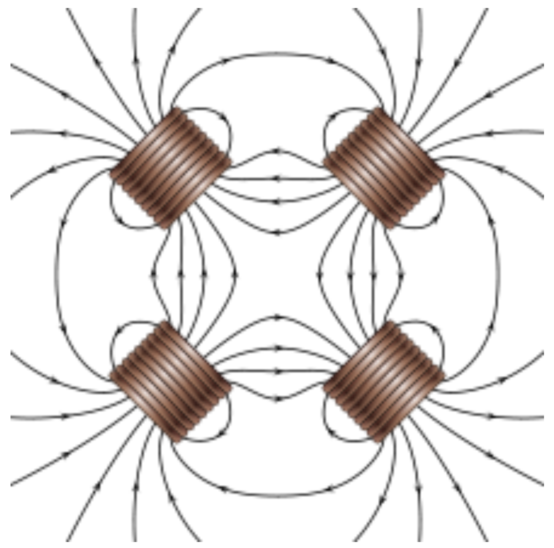
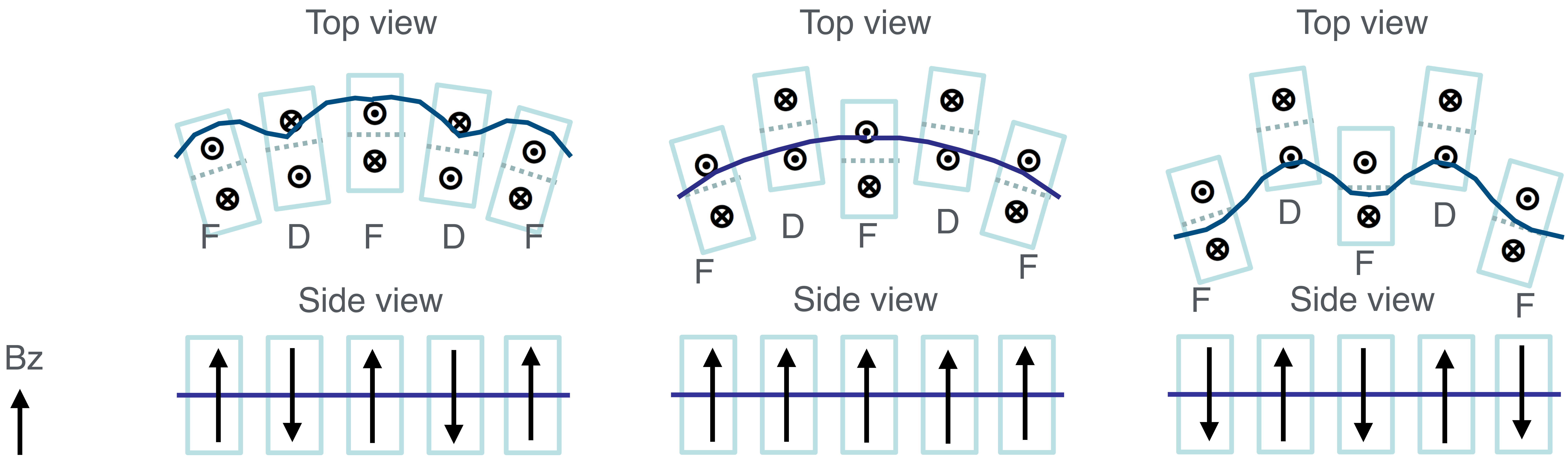
Radial shift of normal quadrupole FODO

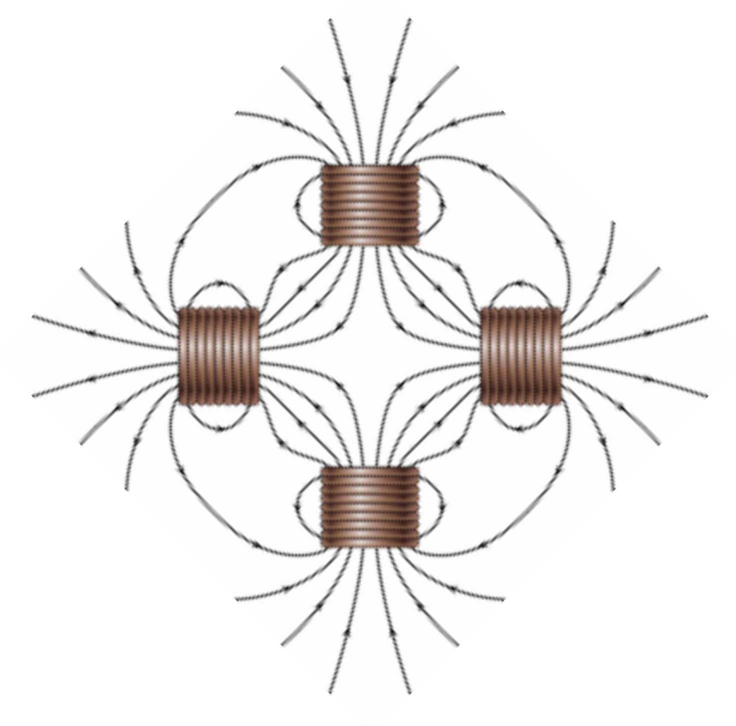


- Beta functions are the same.
- Dispersion function Dx can be small in ns-FFA lattice configuration.
- **Dispersion action function H** is minimum in ns-FFA so that momentum compaction factor is zero.

Scaling FFA like (large alpha) | **Combined function** | **Non scaling FFA (small alpha)**



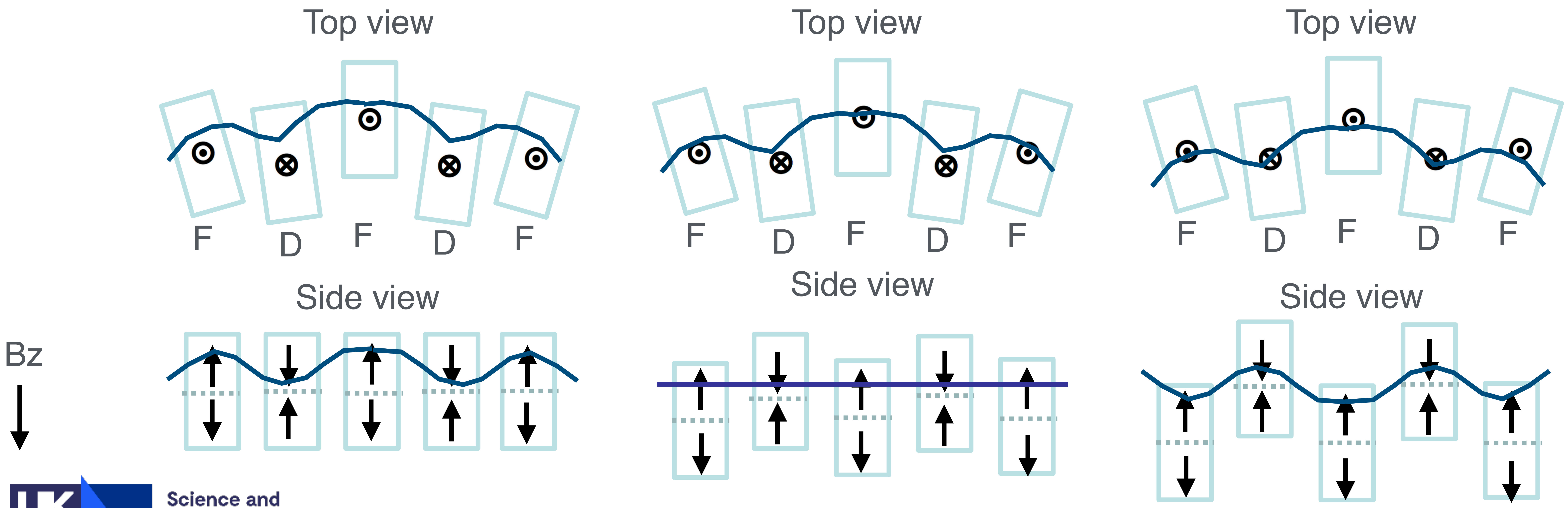
Vertical shift of skew quadrupole FODO



- Beta functions are the same.
- Dispersion function Dy can be small with larger shift.
- **Dispersion action function H** is minimum with small Dy so that momentum compaction factor is zero.

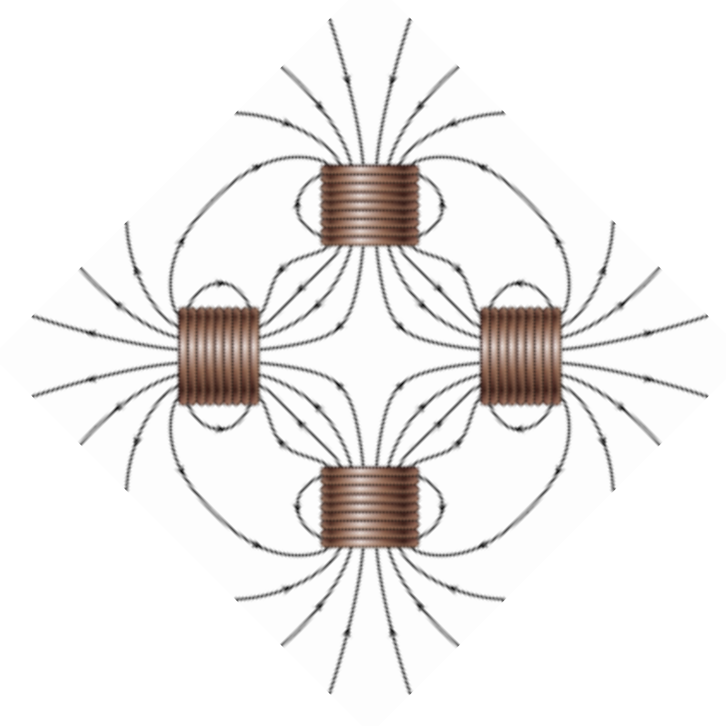
Scaling vFFA like (large alpha)

with larger shift (small alpha) corresponds to horizontal ns-FFA

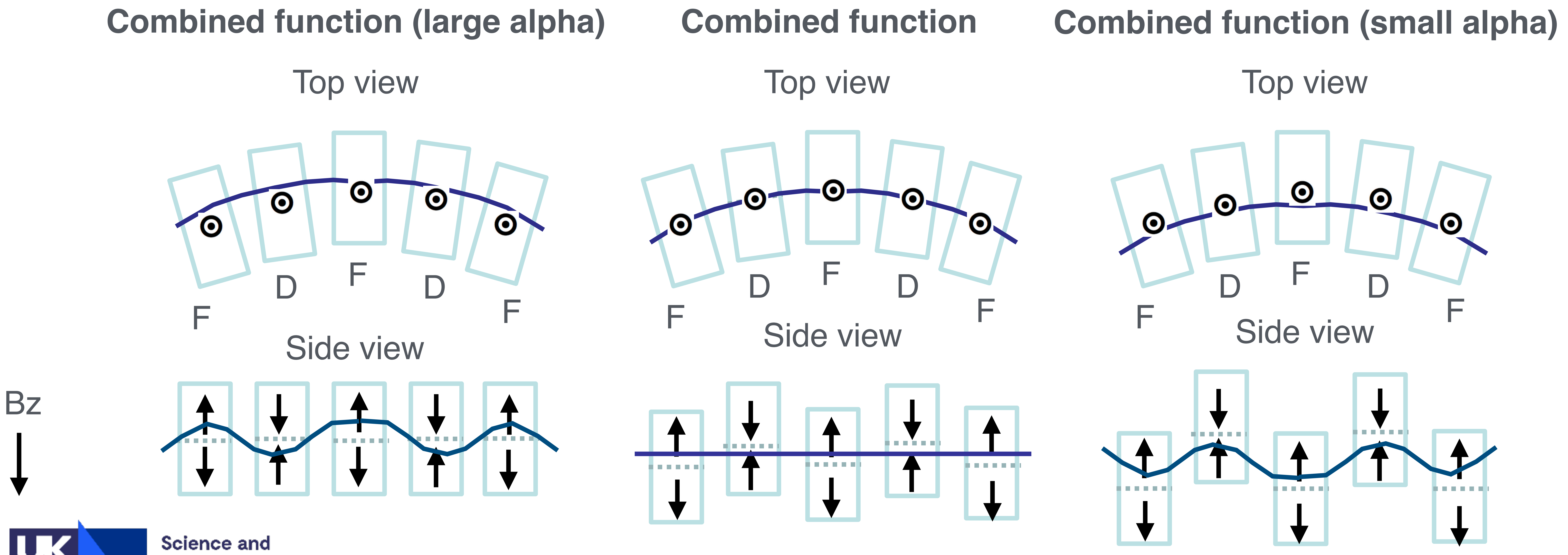


Vertical shift of skew quadrupole FODO

without reverse bend

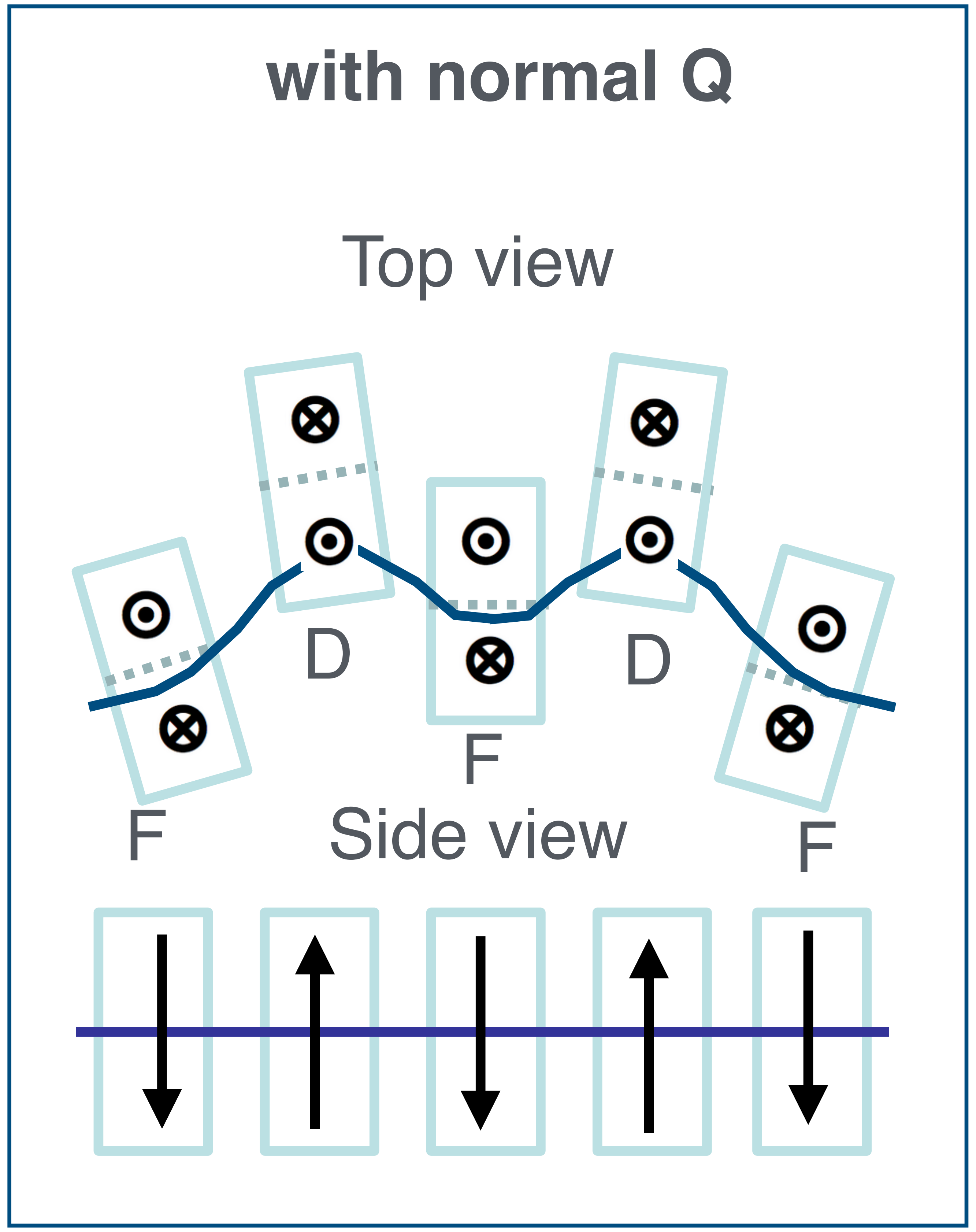


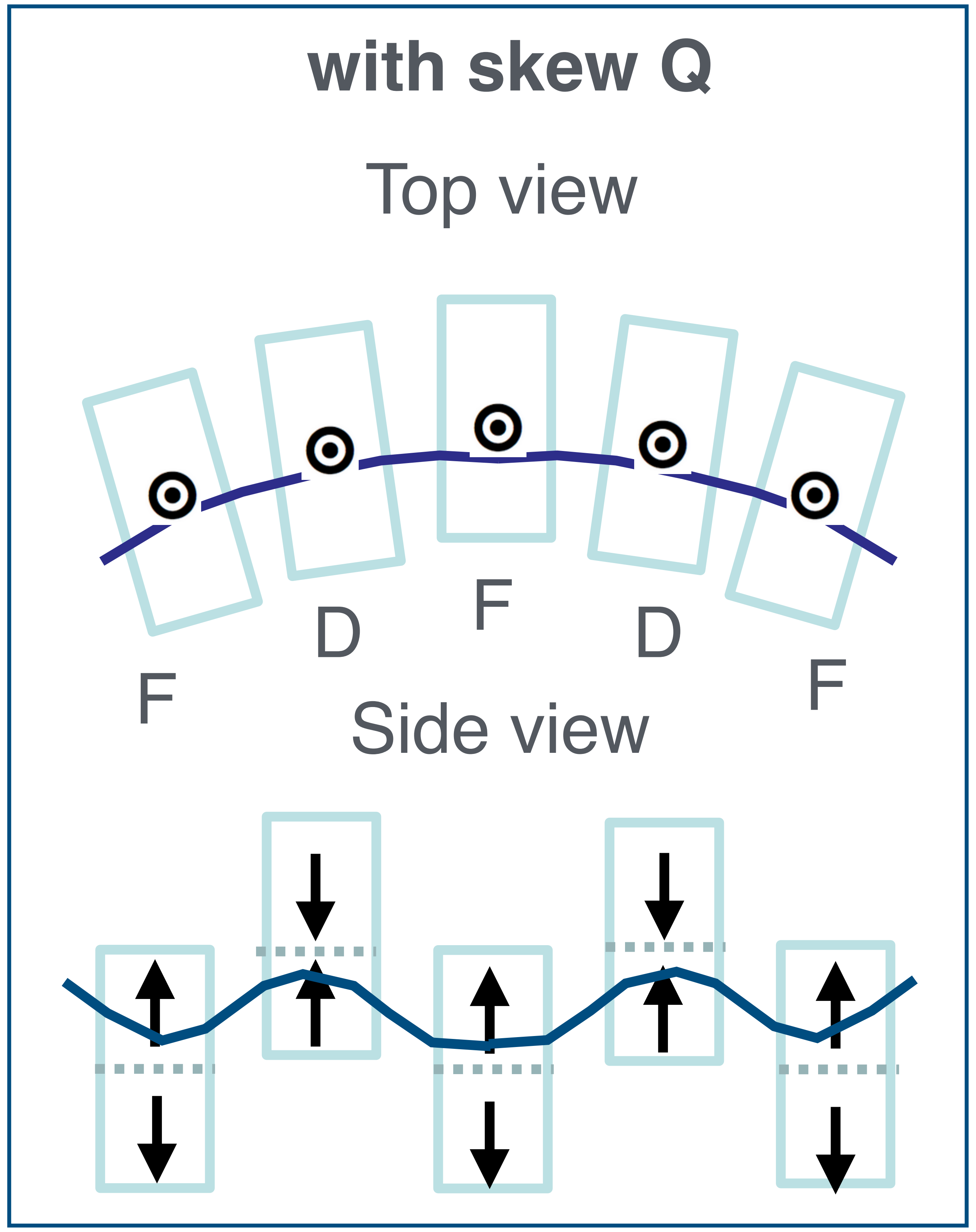
- Unlike vFFA, orbit could go below the mid-plane of skew Q where the sign of vertical field flipped.
- We can eliminate reverse bend.



1.5 TeV collider ring

Impose constraint of momentum compaction (alpha)= 0.





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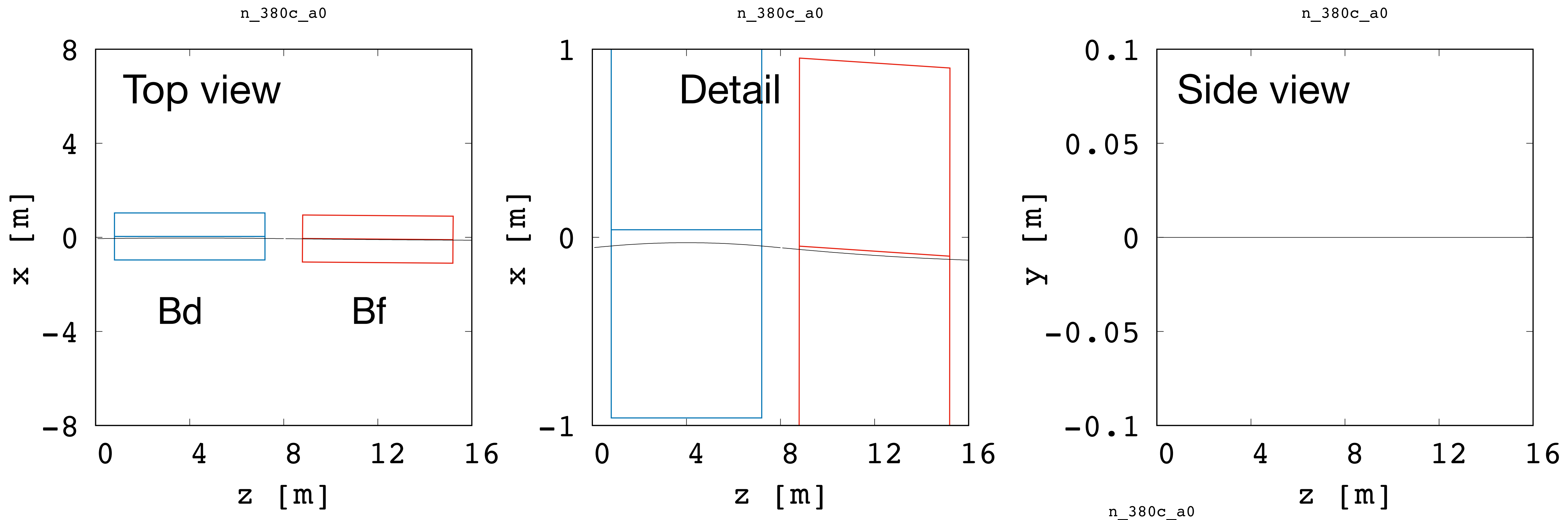
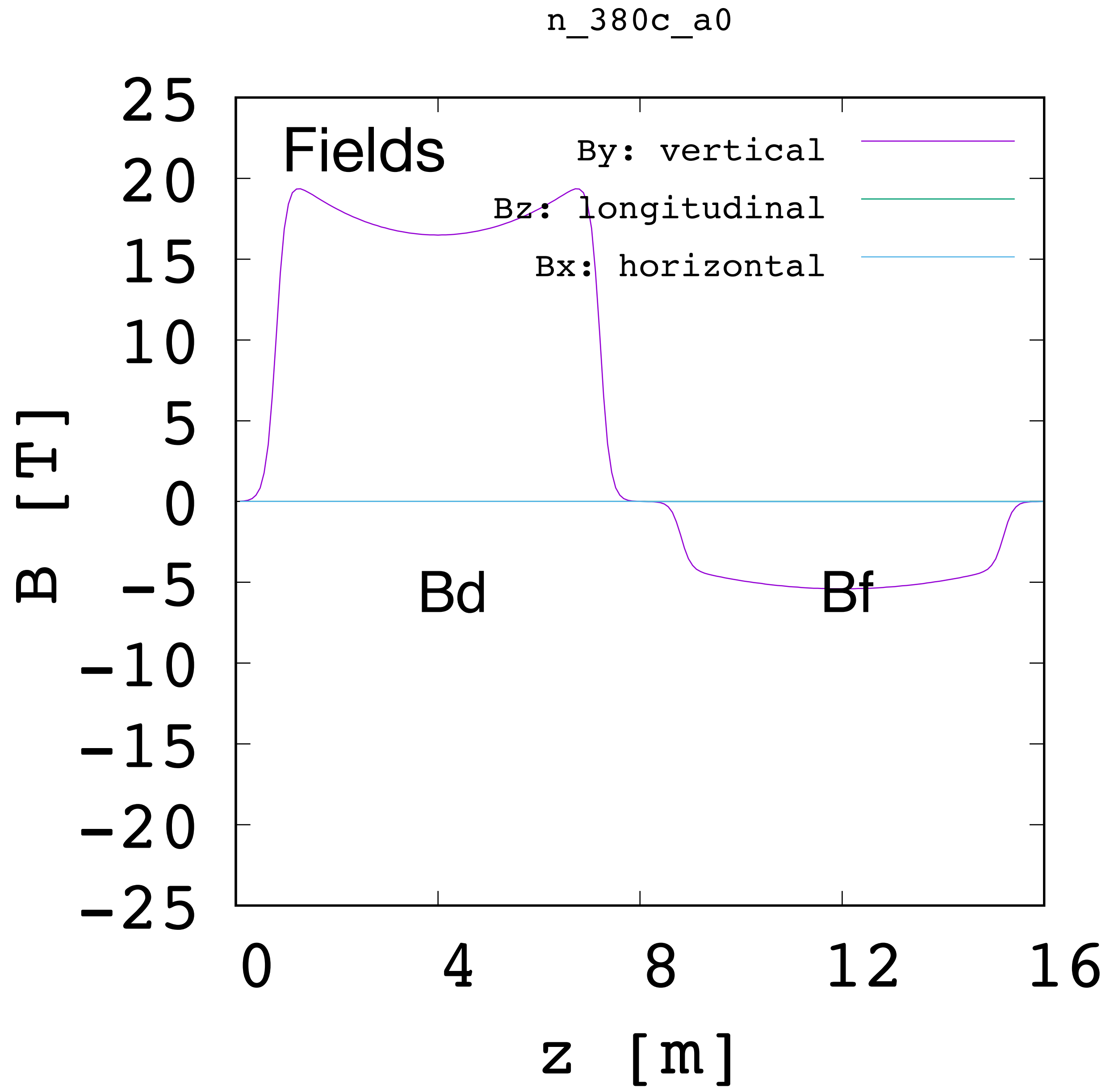
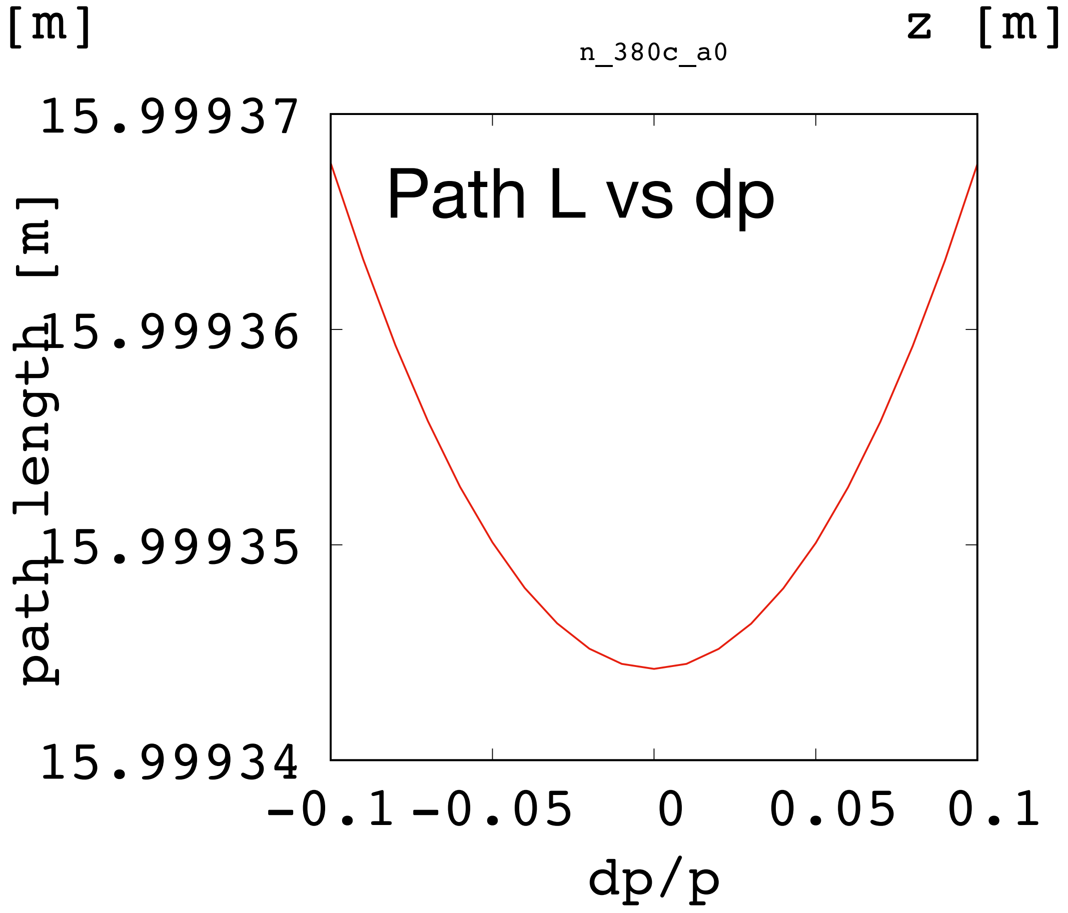
1.5 TeV collider ring

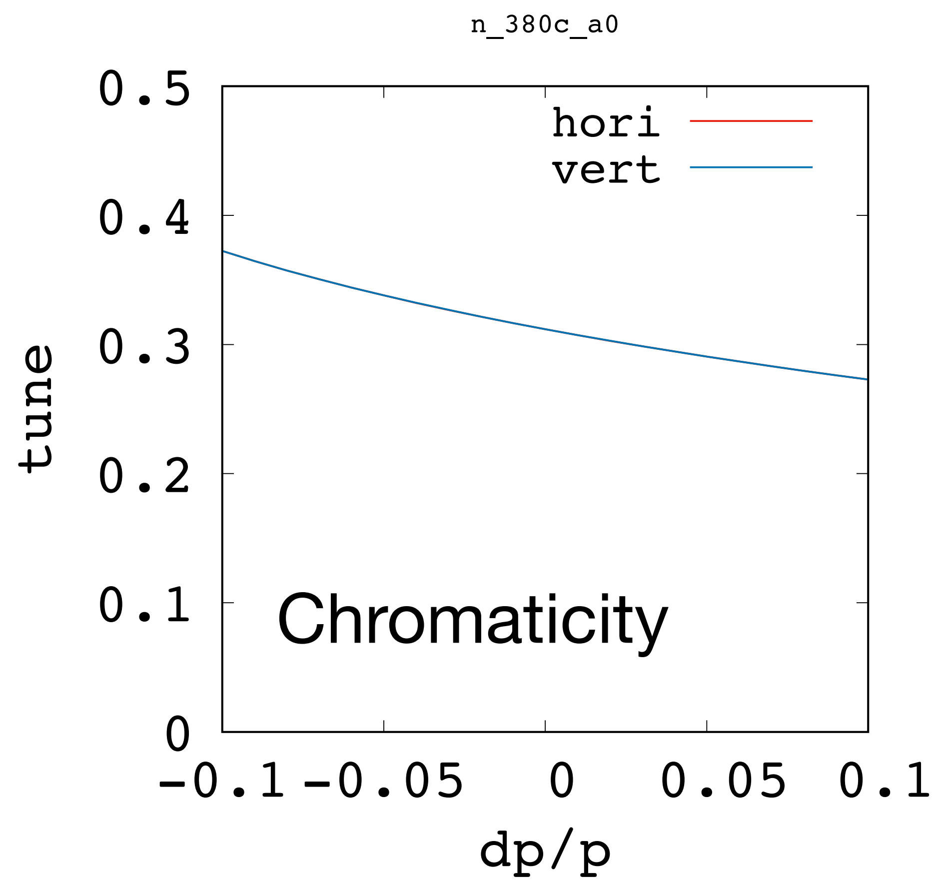
momentum comp=0, arc only

	Normal FODO
Energy	1.5 TeV
Momentum compaction	0
Circumference	6080 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	380
Maximum field	20 T
Field gradient	240 T/m
Cell tune	0.3131 / 0.3131









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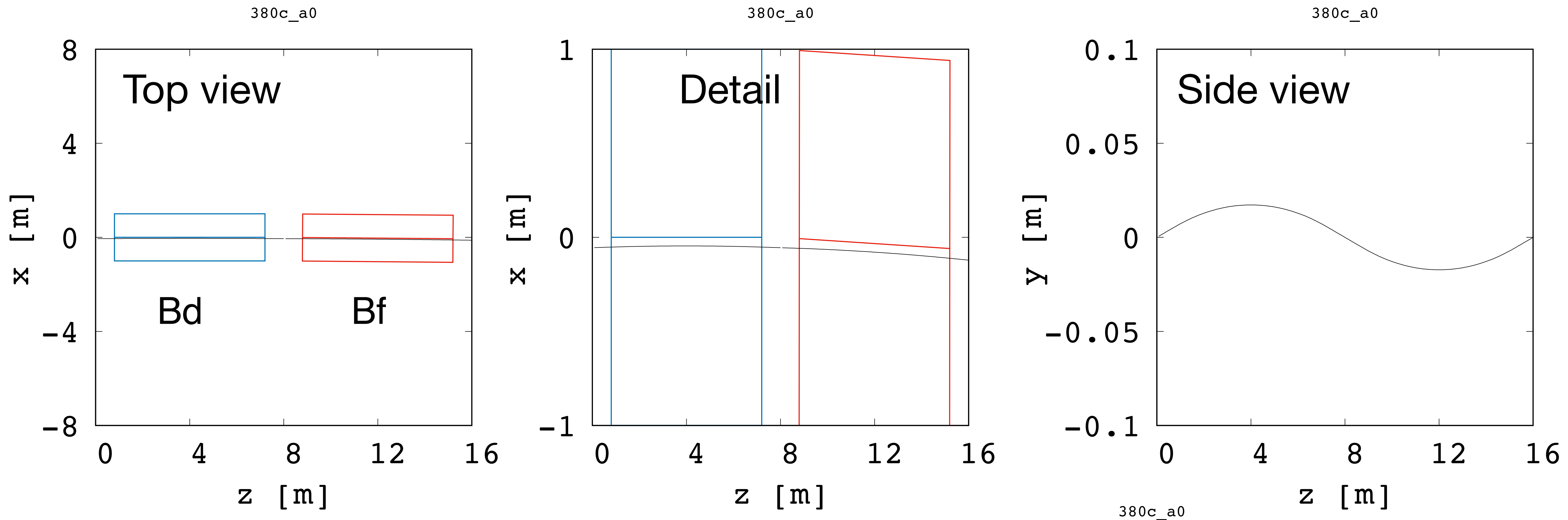
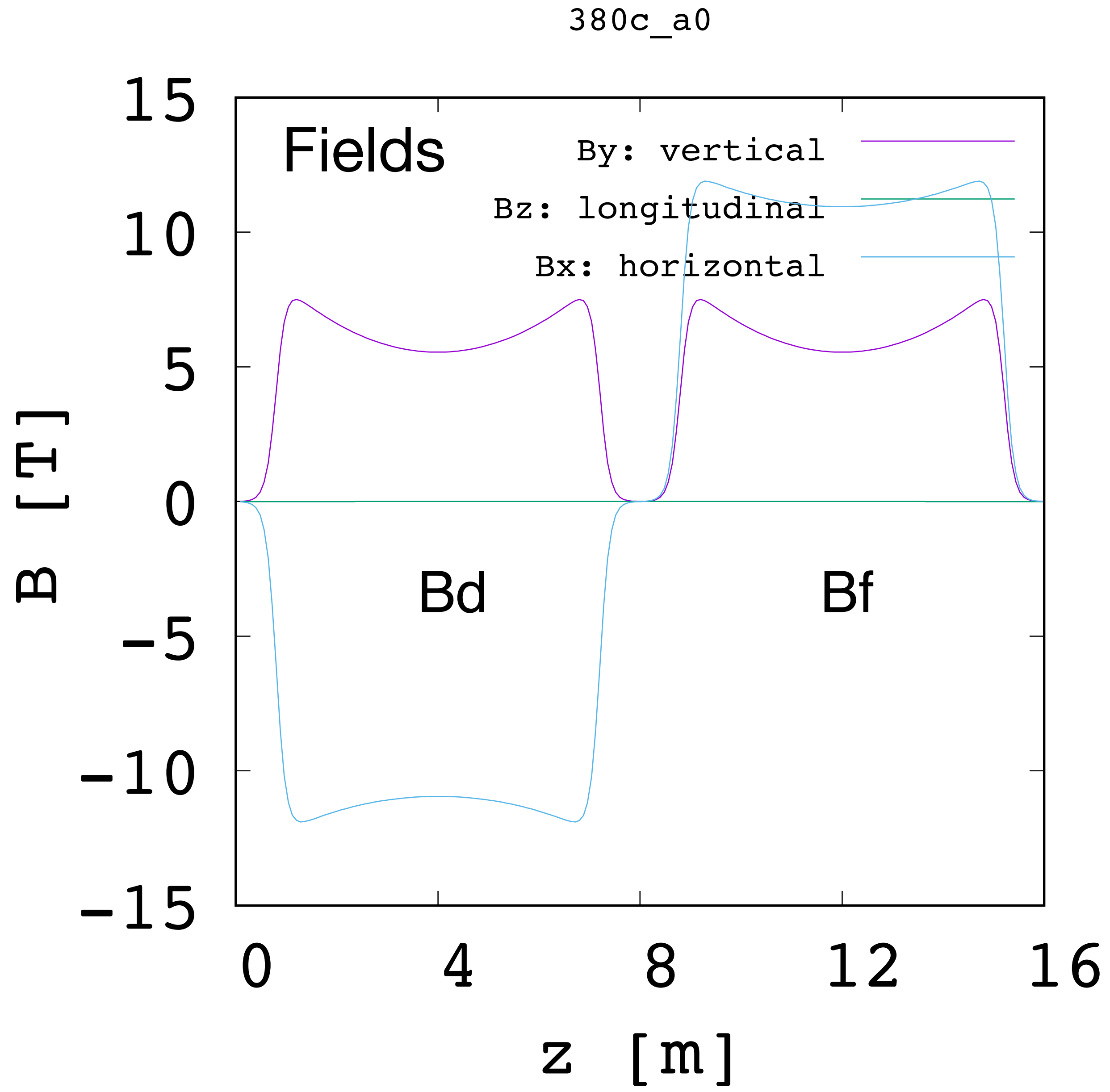
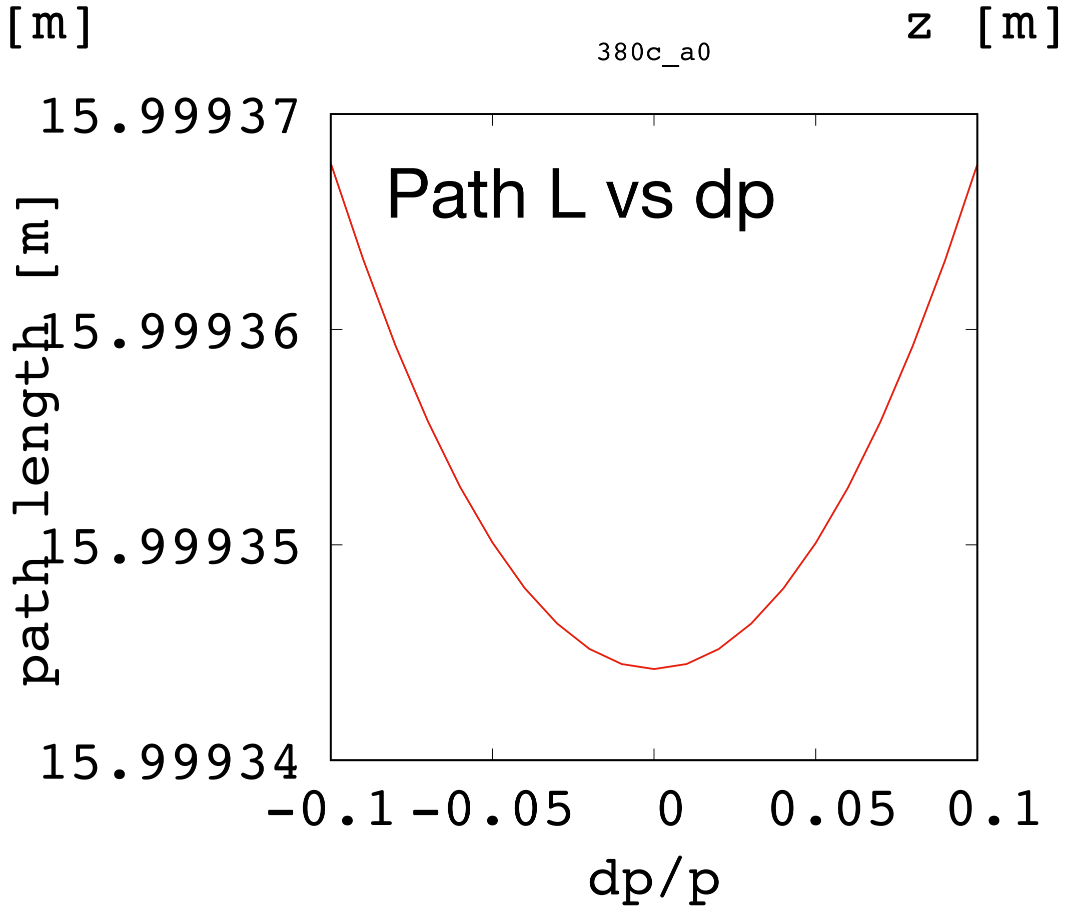
1.5 TeV collider ring

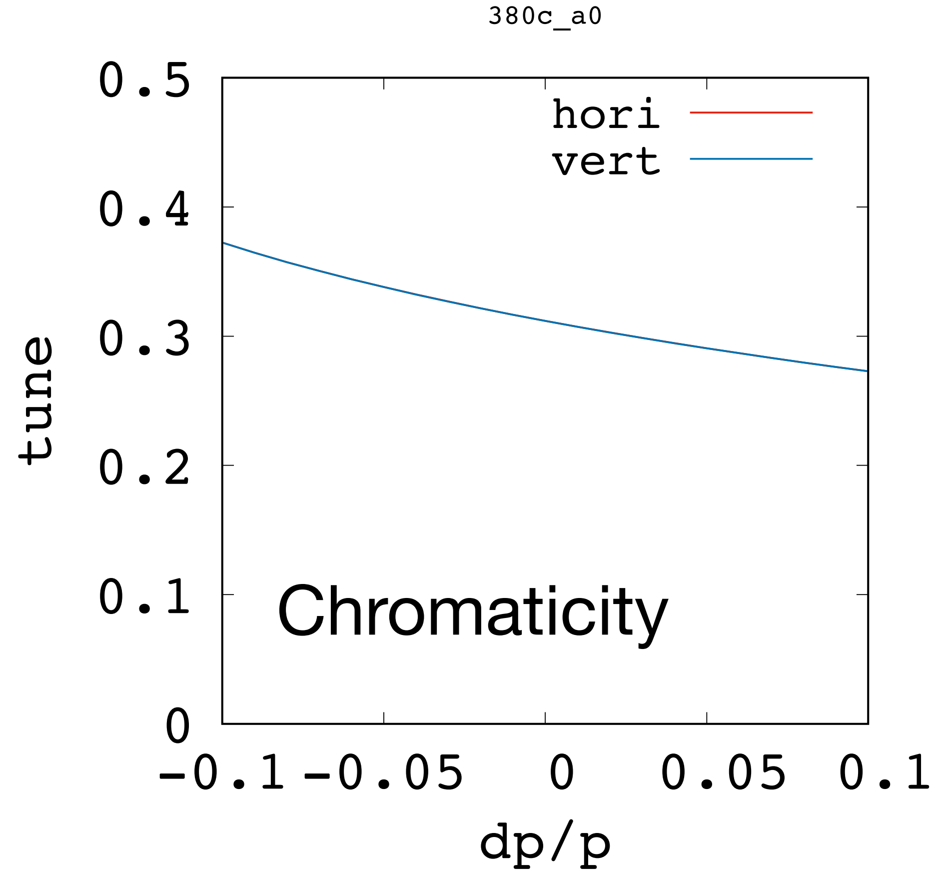
momentum comp=0, arc only

	Skew FODO
Energy	1.5 TeV
Momentum compaction	0
Circumference	6080 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	380
Maximum field	14 T
Field gradient	240 T/m
Cell tune	0.3131 / 0.3131









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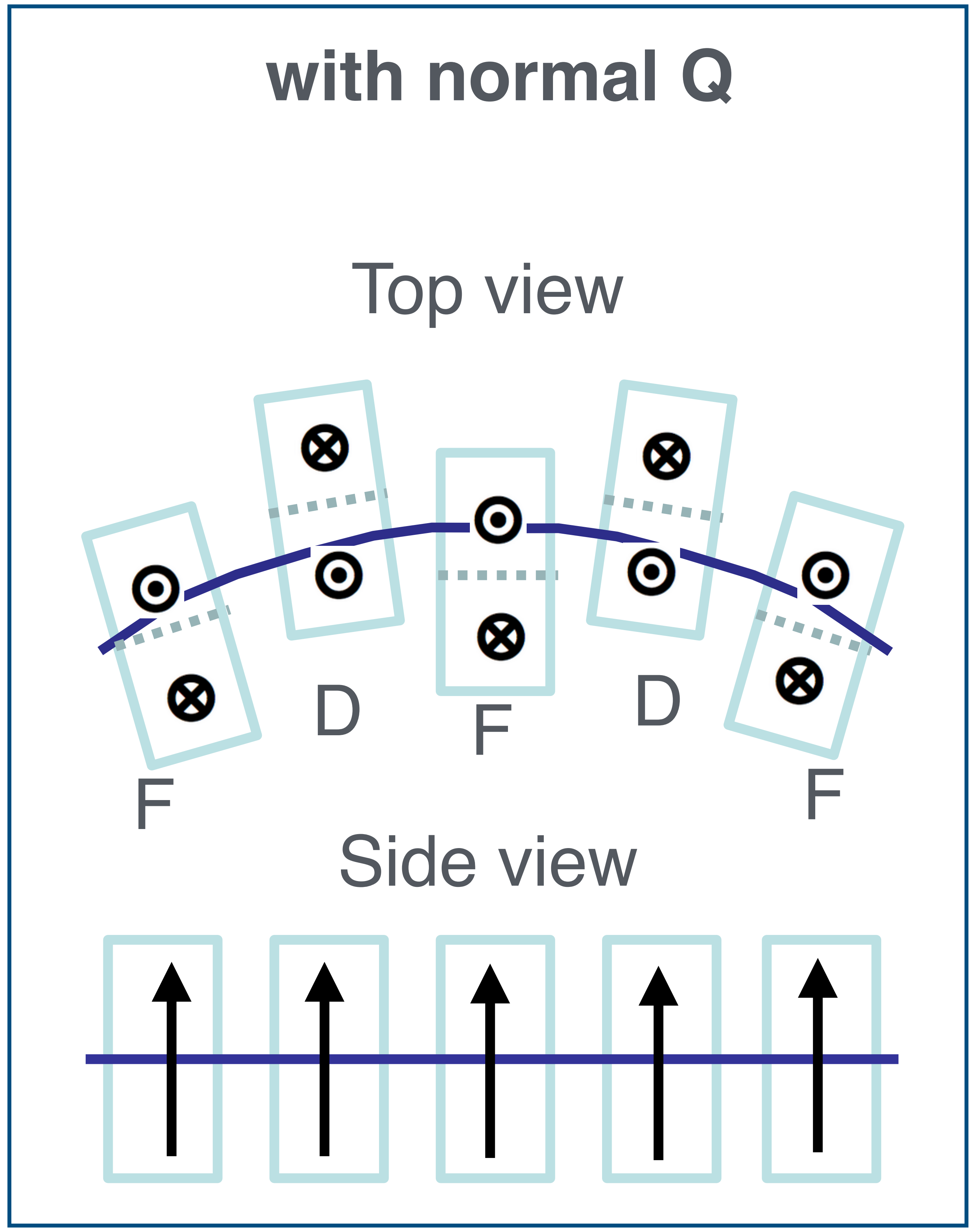
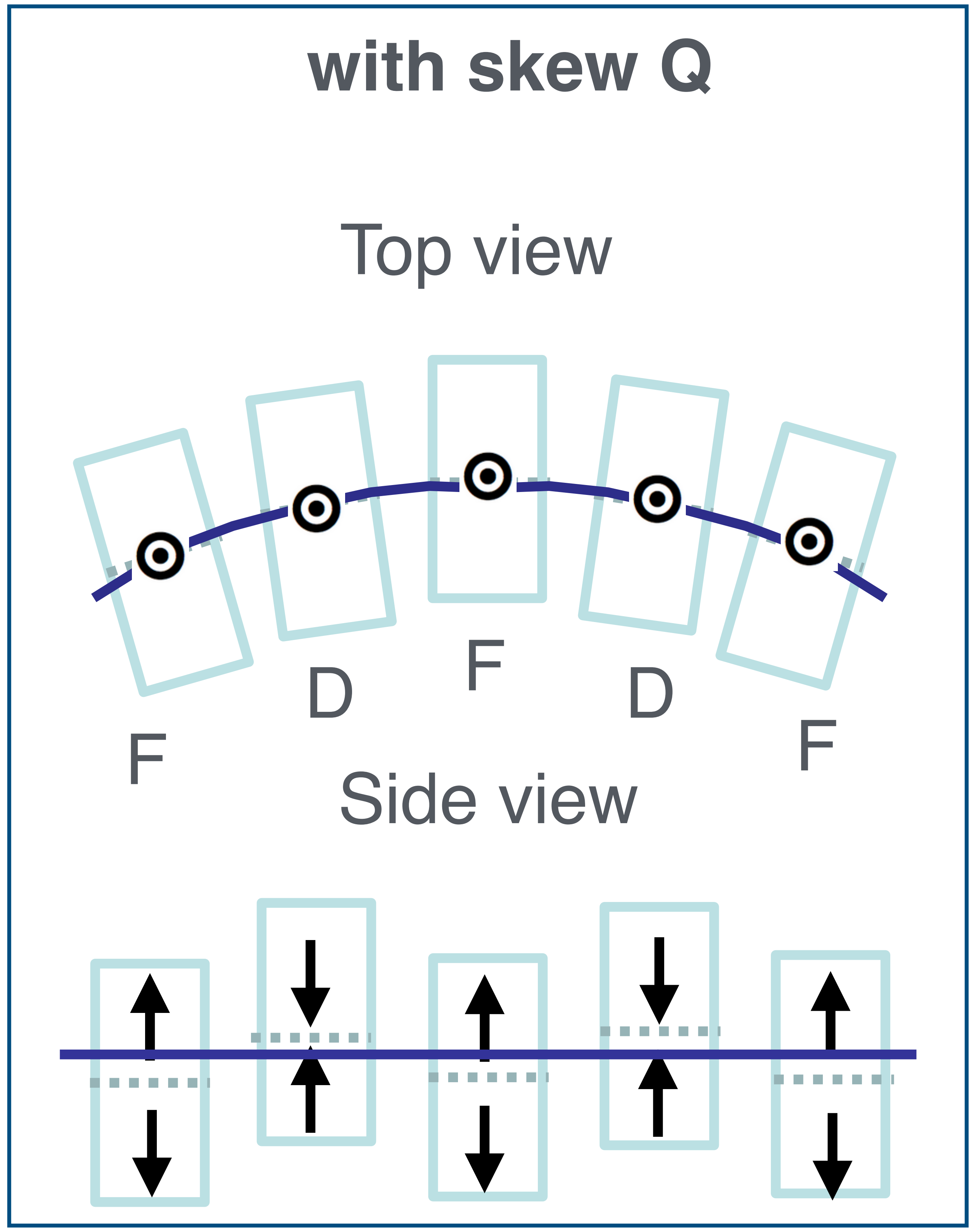
1.5 TeV collider ring

Without constraint of momentum compaction (alpha)= 0.

Minimise circumference.





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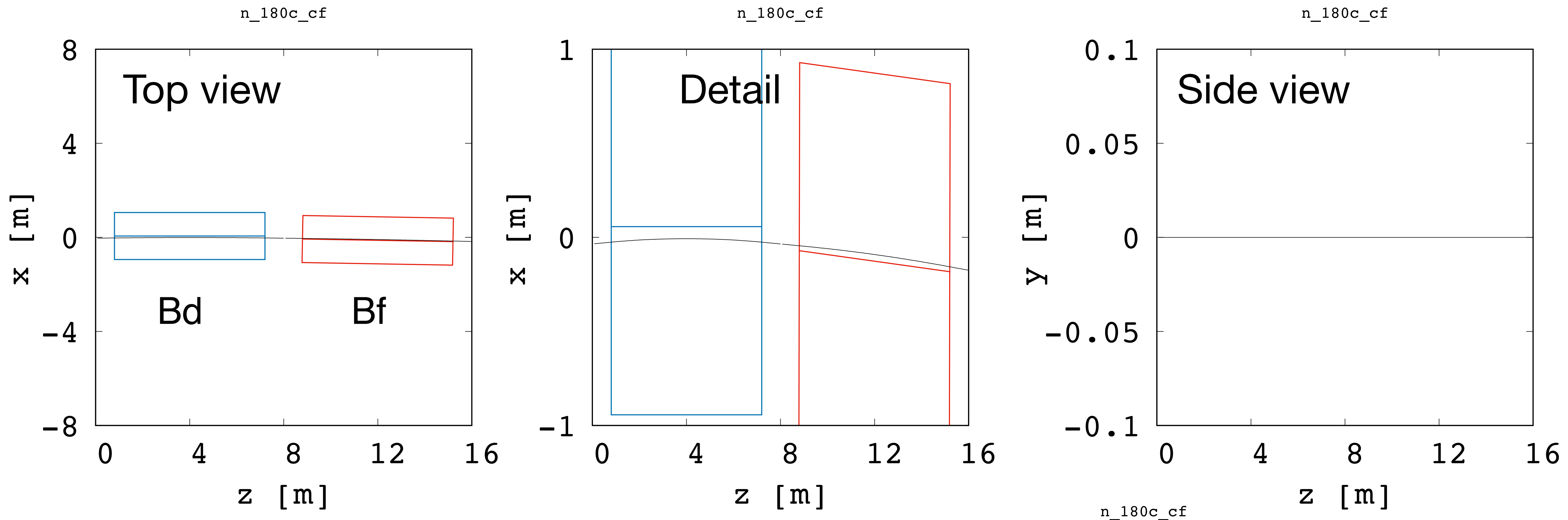
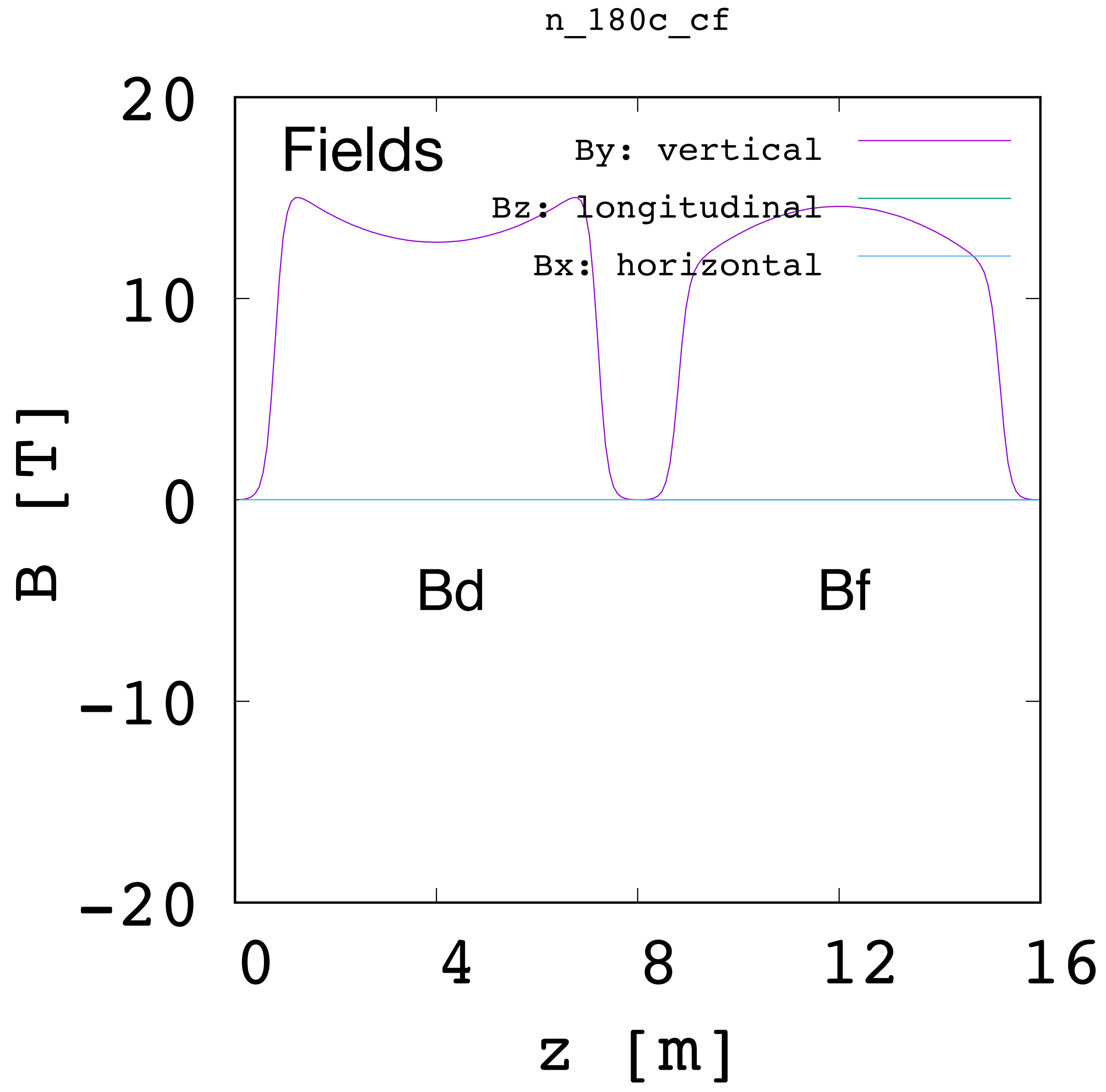
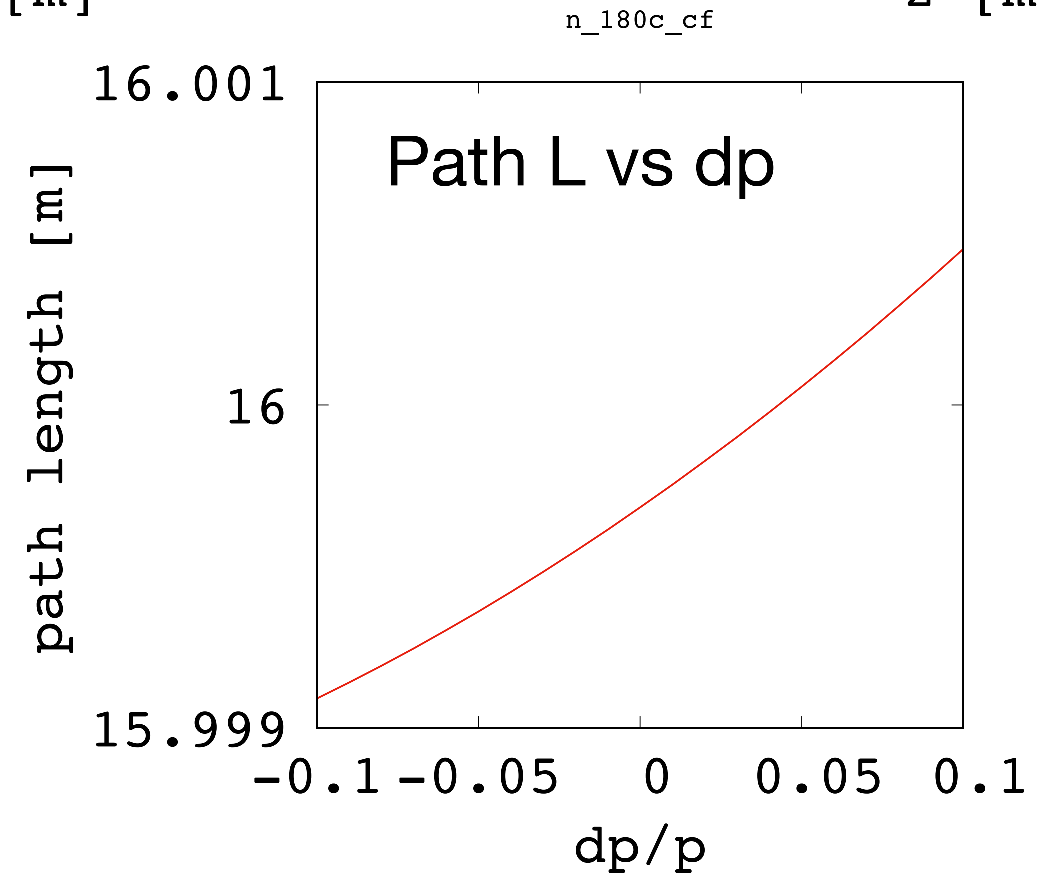
1.5 TeV collider ring

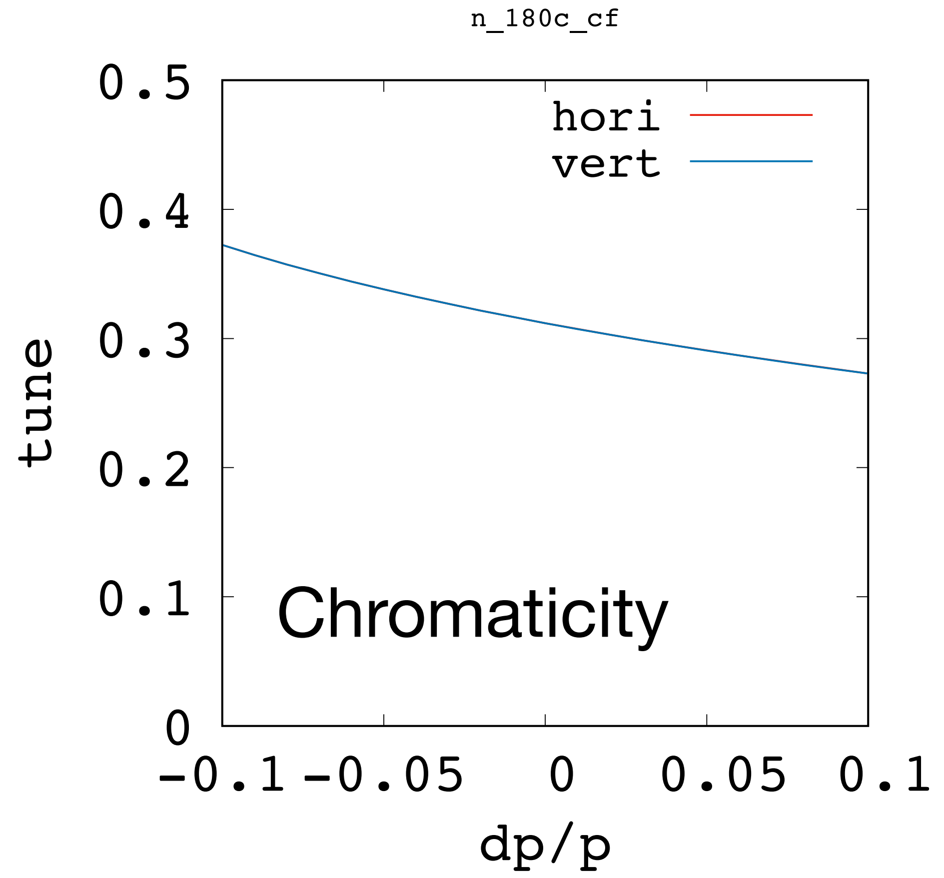
minimise circumference, arc only

	Normal FODO
Energy	1.5 TeV
Momentum compaction	4.32 x 10^{-4}
Circumference	2880 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	180
Maximum field	14 T
Field gradient	240 T/m
Cell tune	0.3119 / 0.3119









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1.5 TeV collider ring

minimise circumference, arc only

	Skew FODO
Energy	1.5 TeV
Momentum compaction	4.32 x 10^{-4}
Circumference	2880 m
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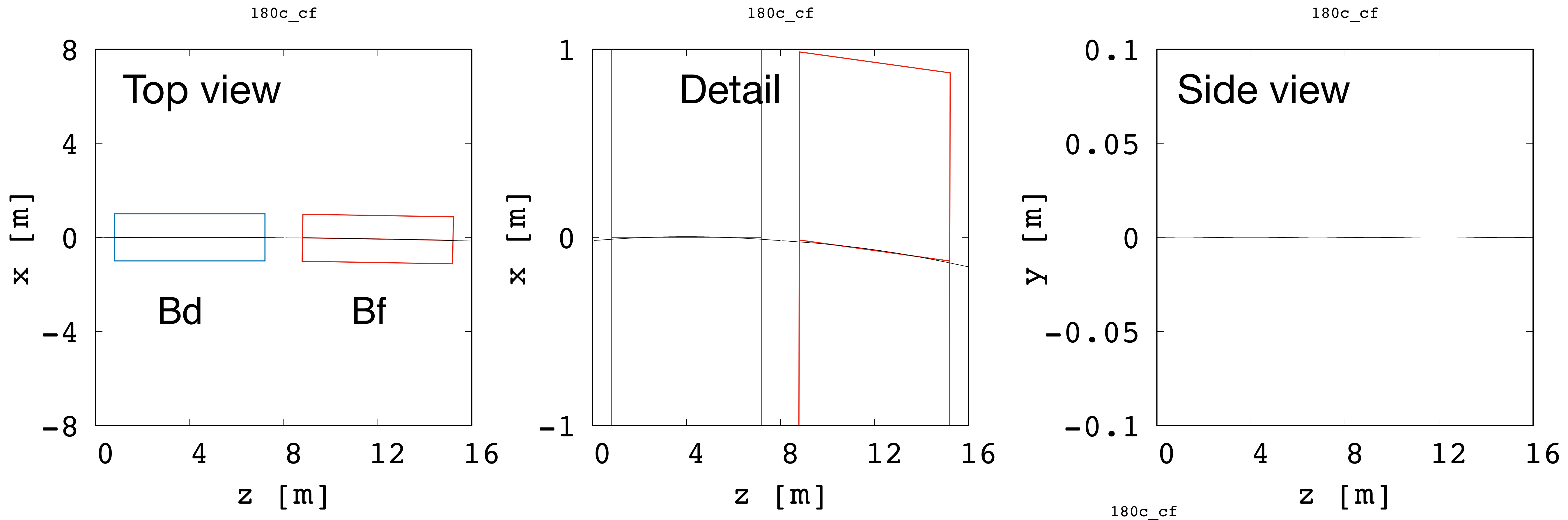

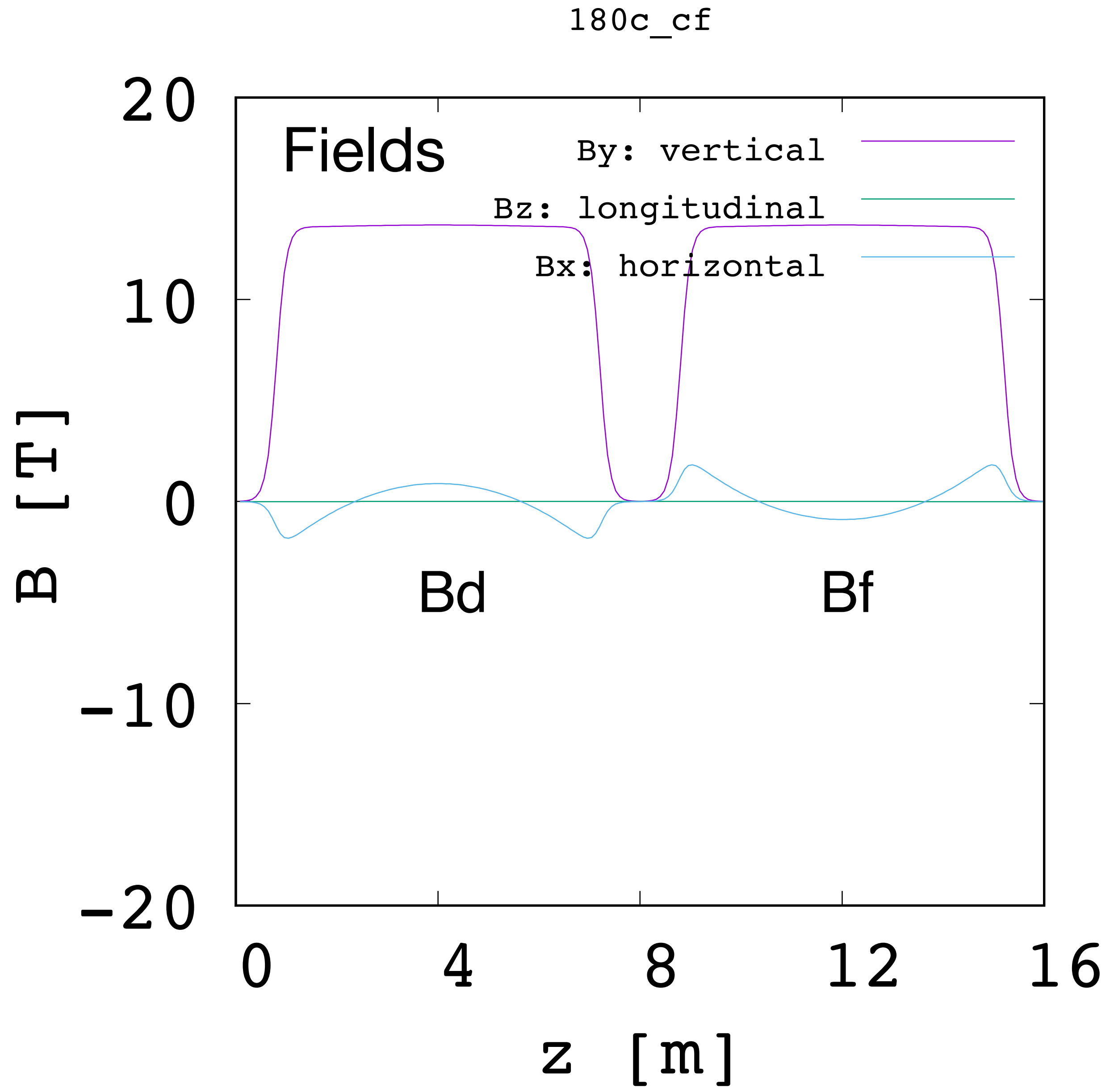

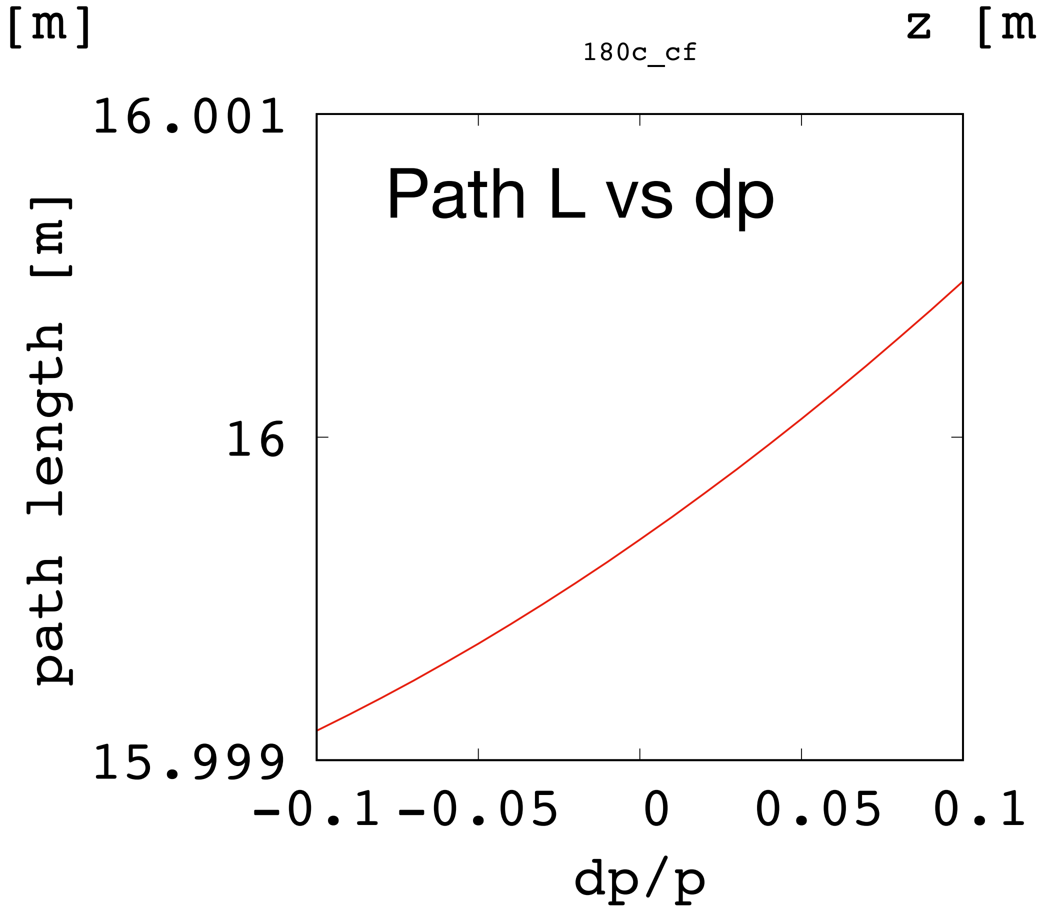

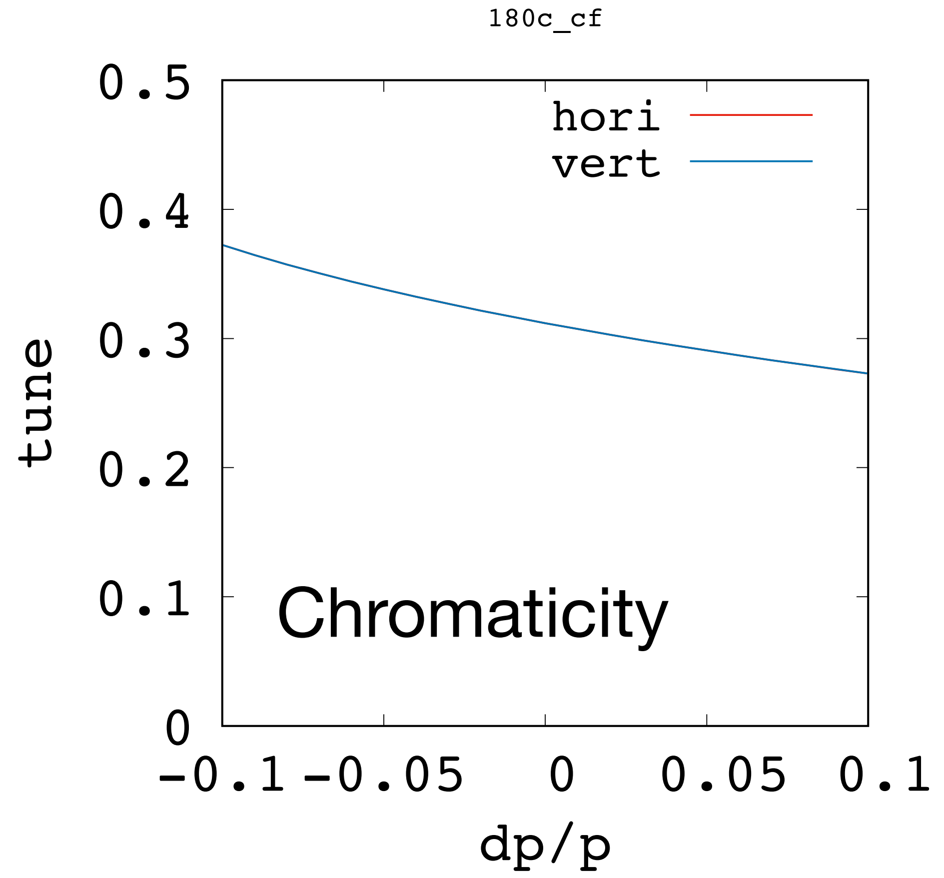

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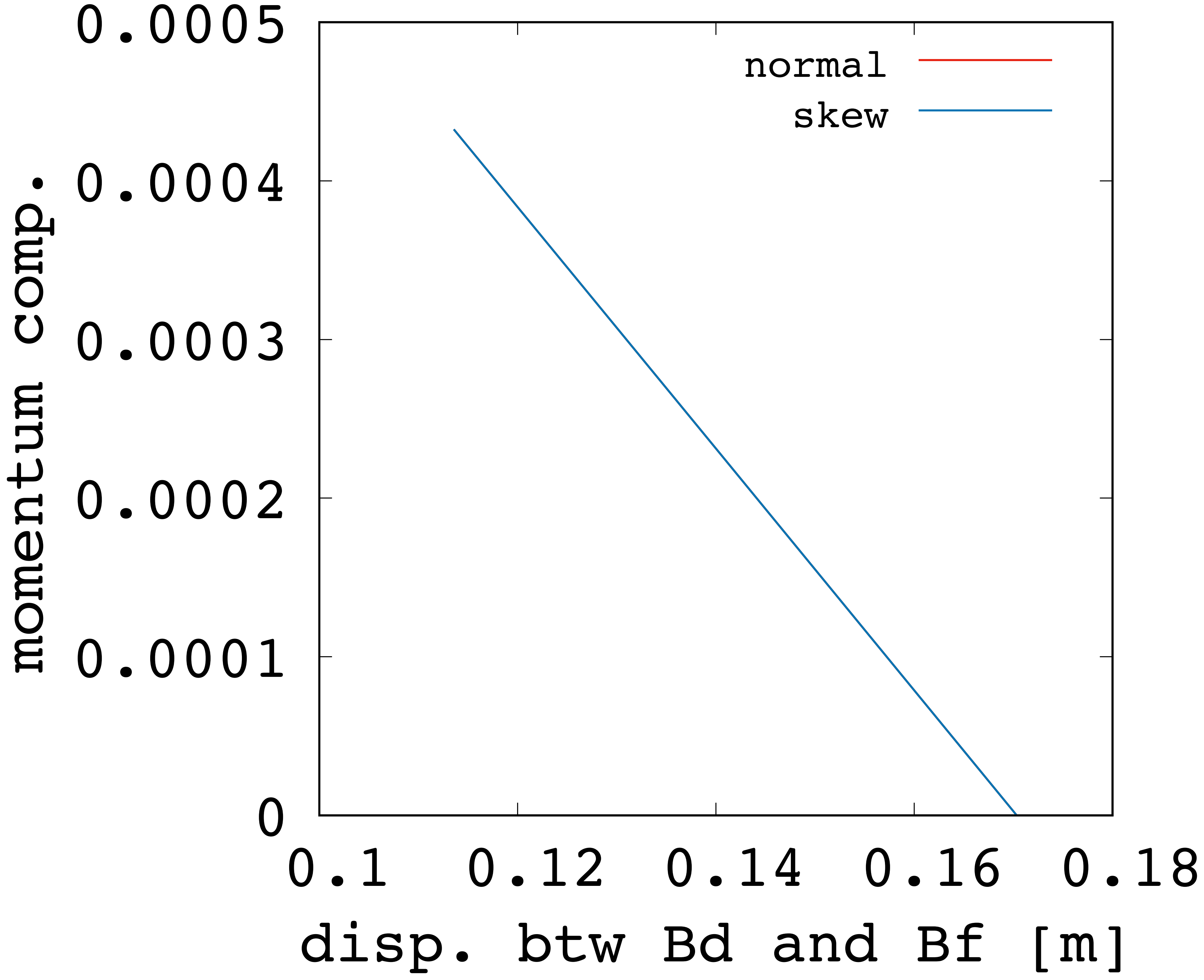
Summary

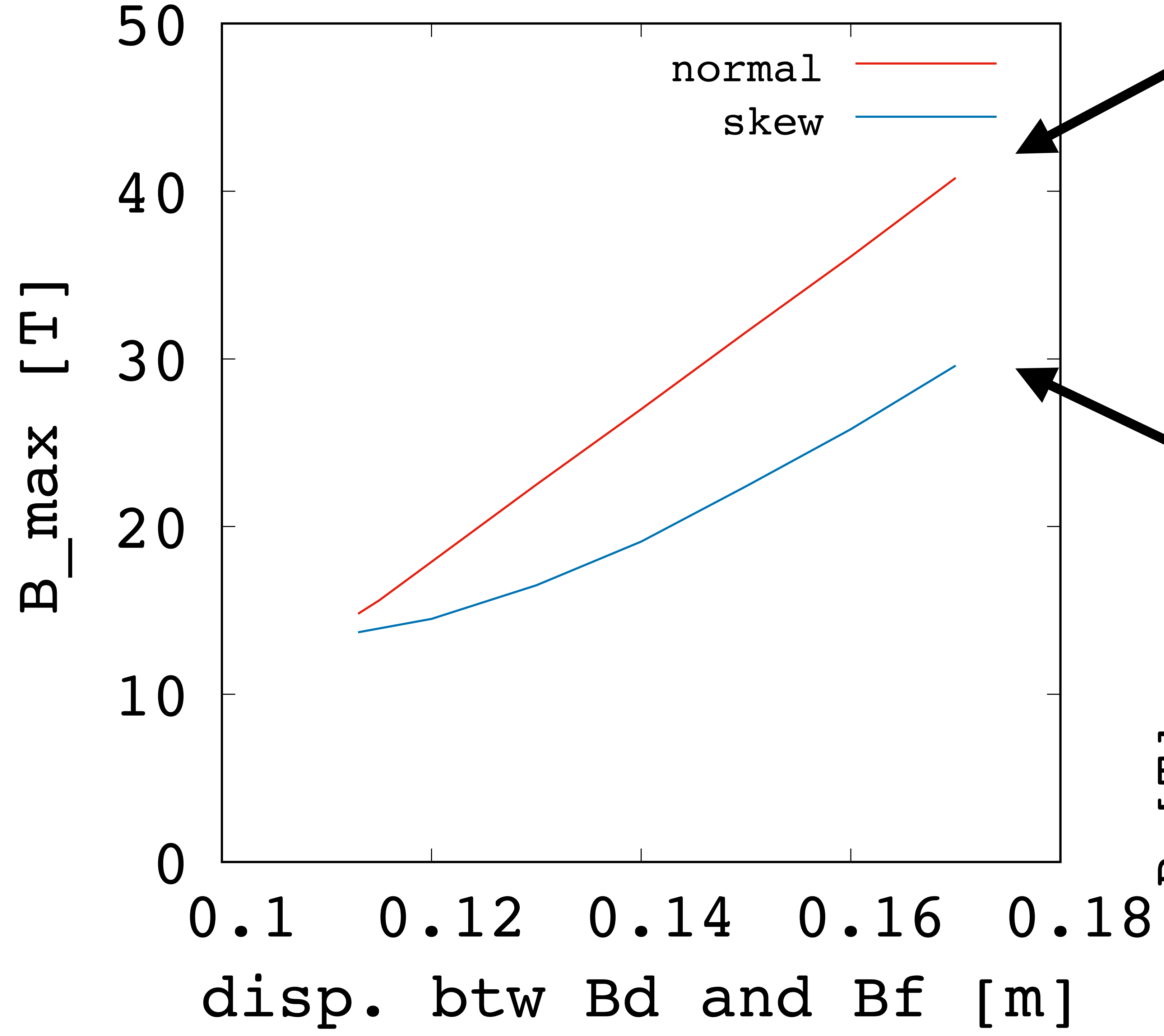
- vFFA for muon accelerator
  - 1.5 TeV accelerator in LHC tunnel was designed.
  - Reverse bend increases the circumference.
  - Current design needs 8.7 T magnet.

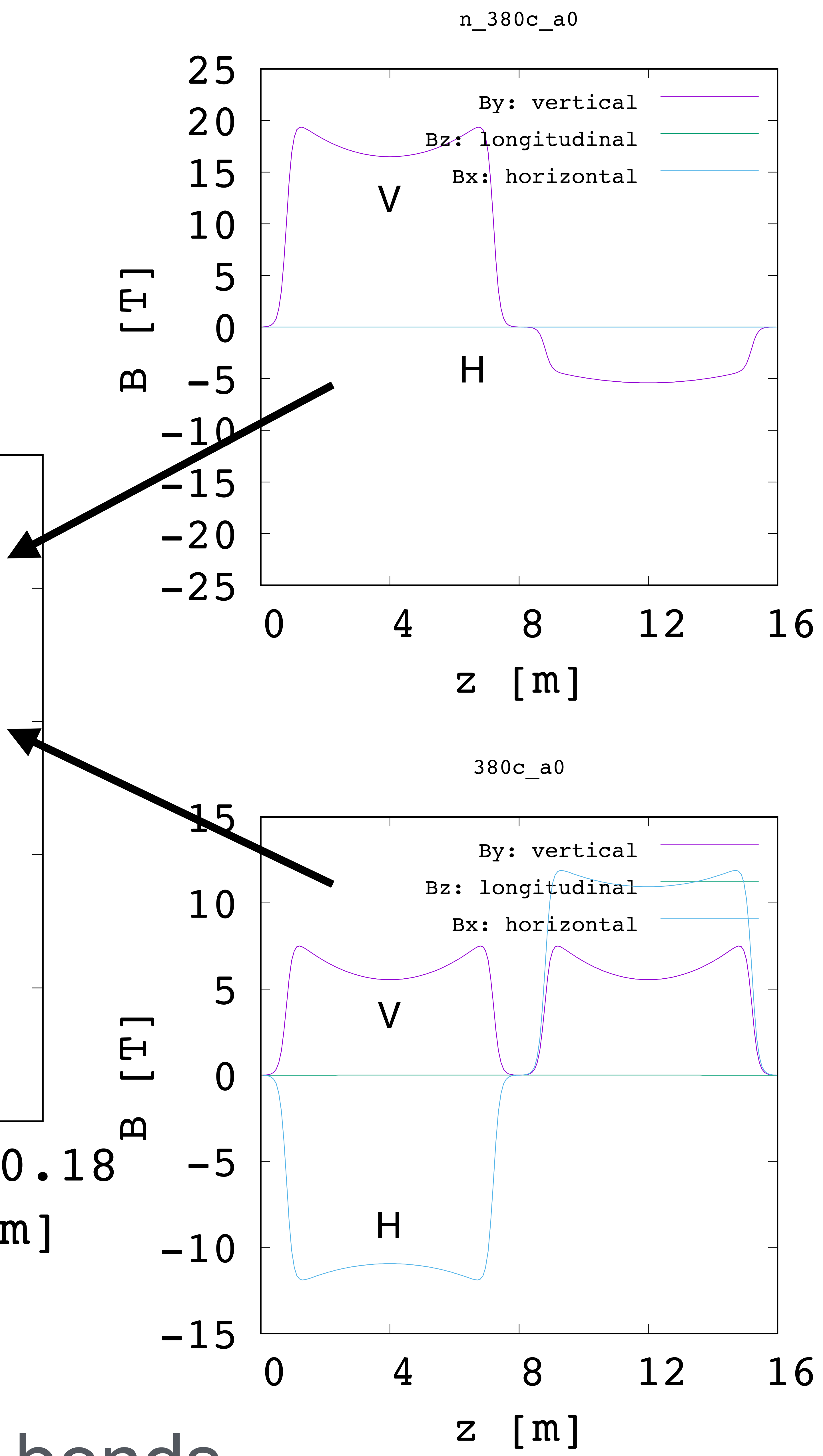
- Muon collider arc design
  - Lattices with normal and skew quadrupoles are classified according to displacement between D and F.
    - FFA like (with reverse bend) vs. combined function lattice (normal bend only)
    - s-FFA type (large dispersion and alpha) vs. ns-FFA type (small dispersion and alpha).
  - Clear advantage on the required magnet strength in skew combined function lattice when momentum compaction factor needs to be minimised.
  - Need low beta insertion design.

1.5 TeV collider ring

momentum compactor and maximum field strength







Skew CF does not need negative bends.

Thank you for your attention

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