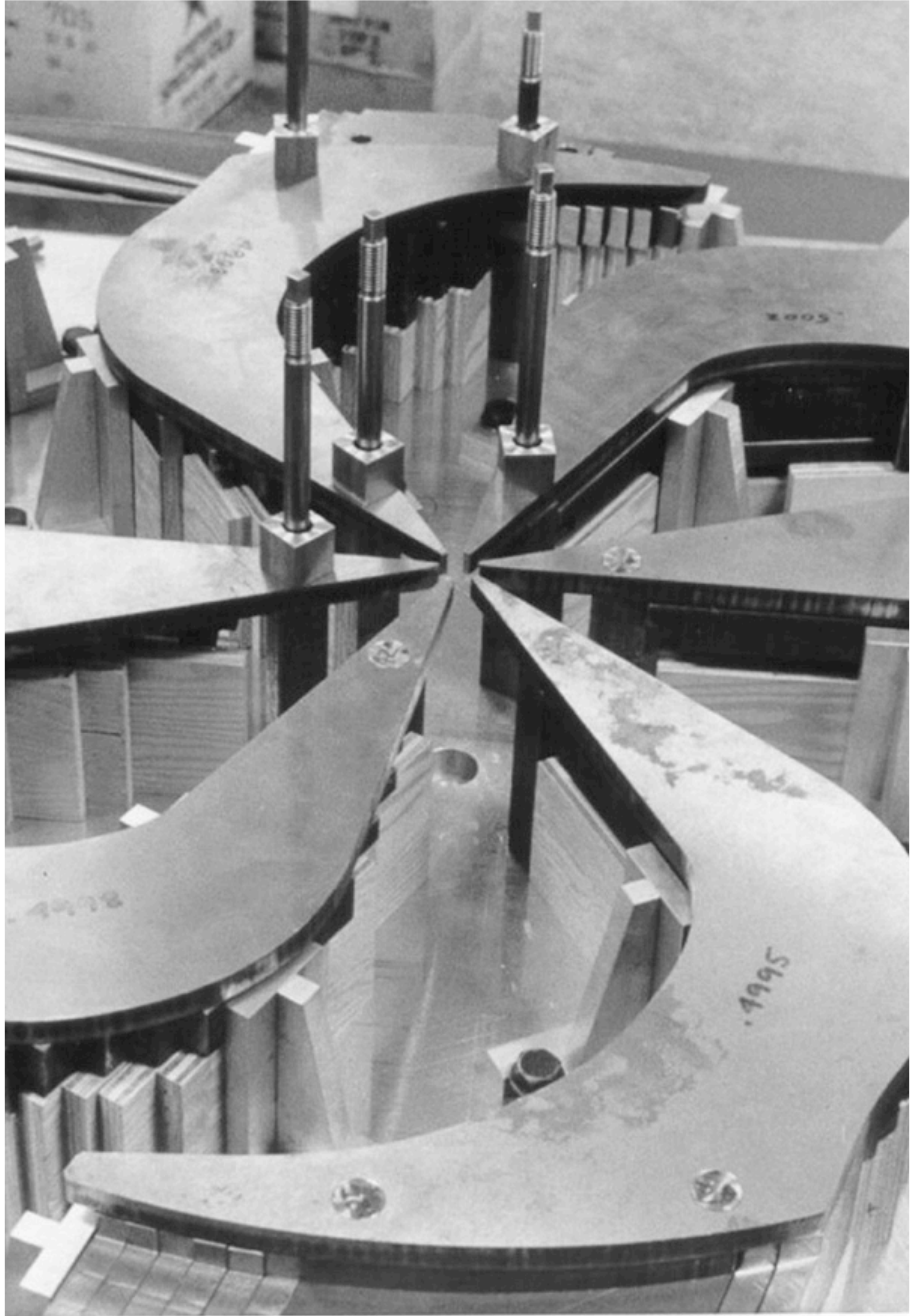
TRIUMF

Feasibility Study for the Cylindrically Symmetric Magnetic Inflector

Lige Zhang

On behalf of
Beam physics group in accelerator division

FFA2020
Dec, 2020



Discovery, accelerated

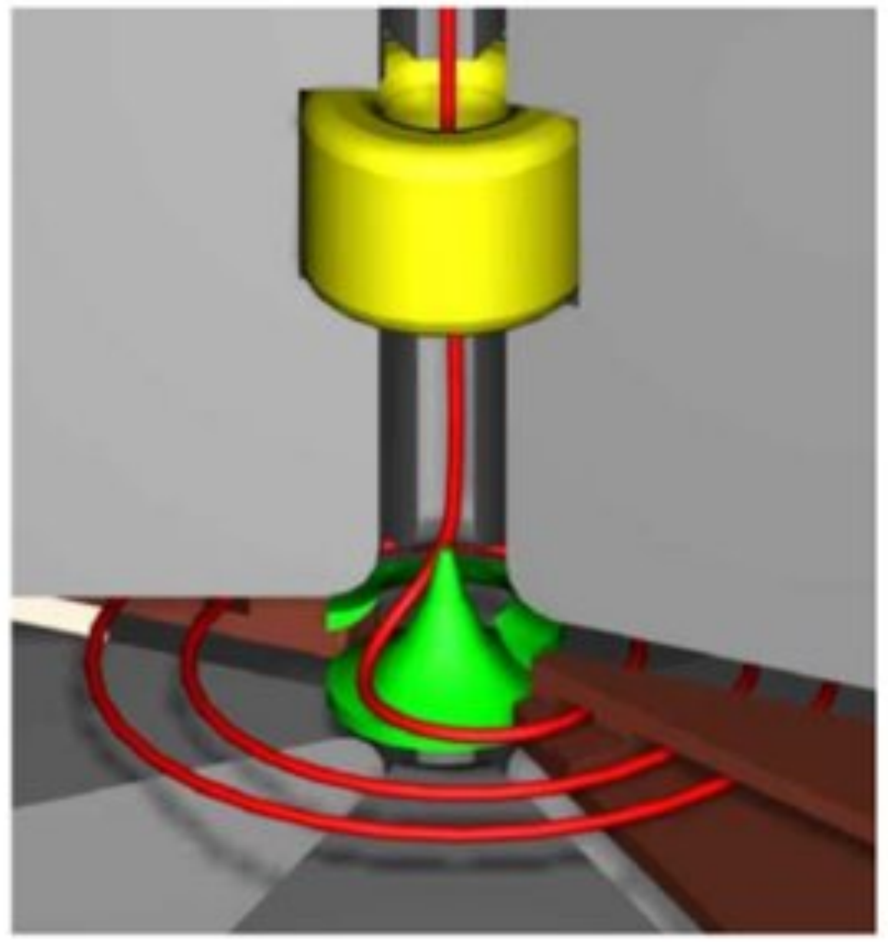
Outline

- Introduction
- Magnetic field and reference orbit
- Beam transport in the moving frame
- Discussion

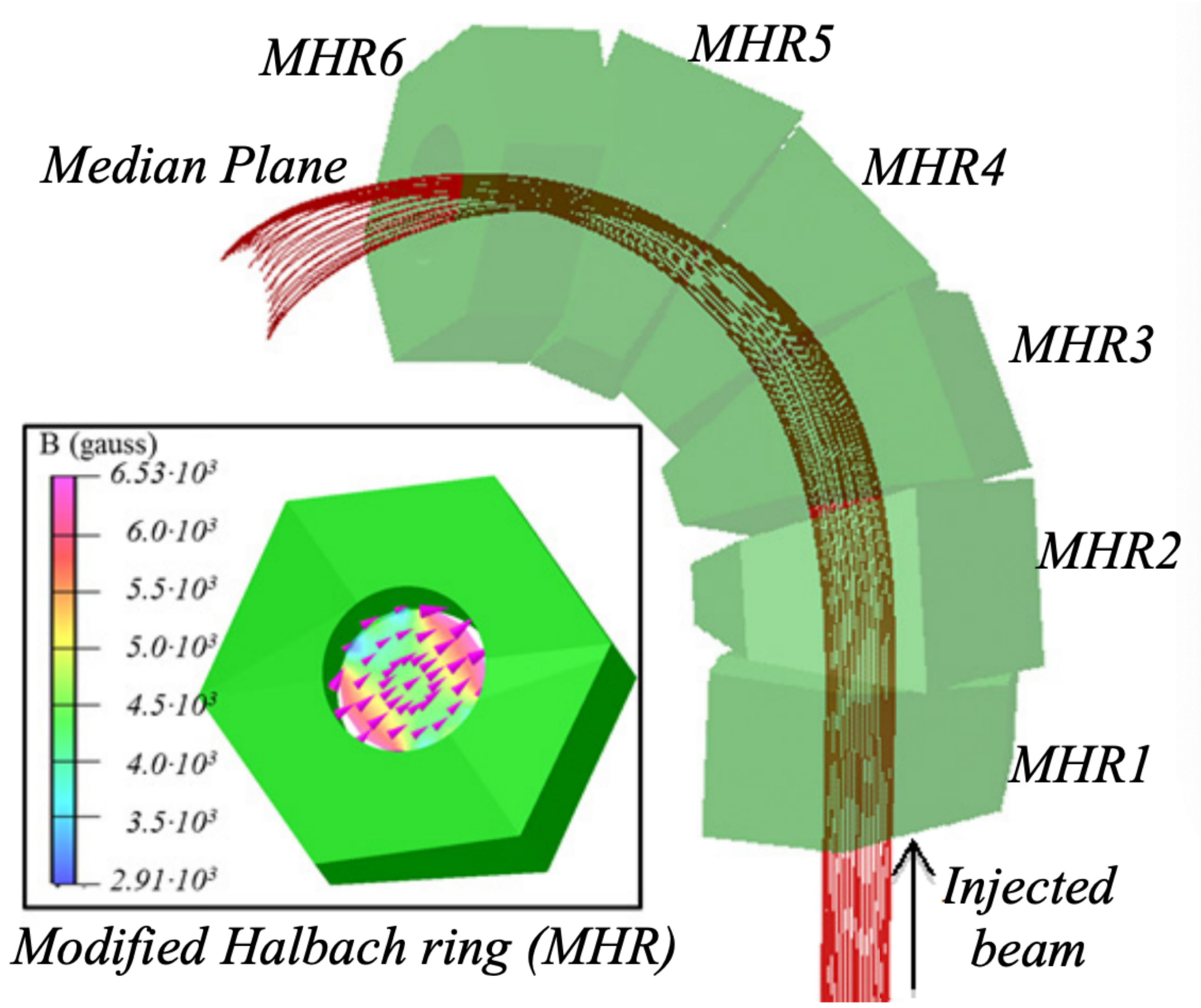
Introduction

- Inflector is used for axial injection into a compact cyclotron
- Conventional electrostatic inflector: injection energy is limited by the breakdown voltage between the 2 spiral electrodes and small aperture limits the injection intensity
- Magnetic inflector could overcome the disadvantages

Passive magnetic inflector using soft iron magnetized by the main magnetic field[1]



Active magnetic inflector using an array of permanent magnets[2]



1. William Kleeven, CAS 2013
2. D. Campo, L. Calabretta & etc., IPAC'14

Constant radial magnetic field

For a cylindrically symmetric field, the magnetic vector potential only compose of the azimuthal component A_θ. We define a vector potential which describes a field with constant B_r

$$A_\theta = \frac{A_0 r_0}{r} + B_0 (z - z_0)$$

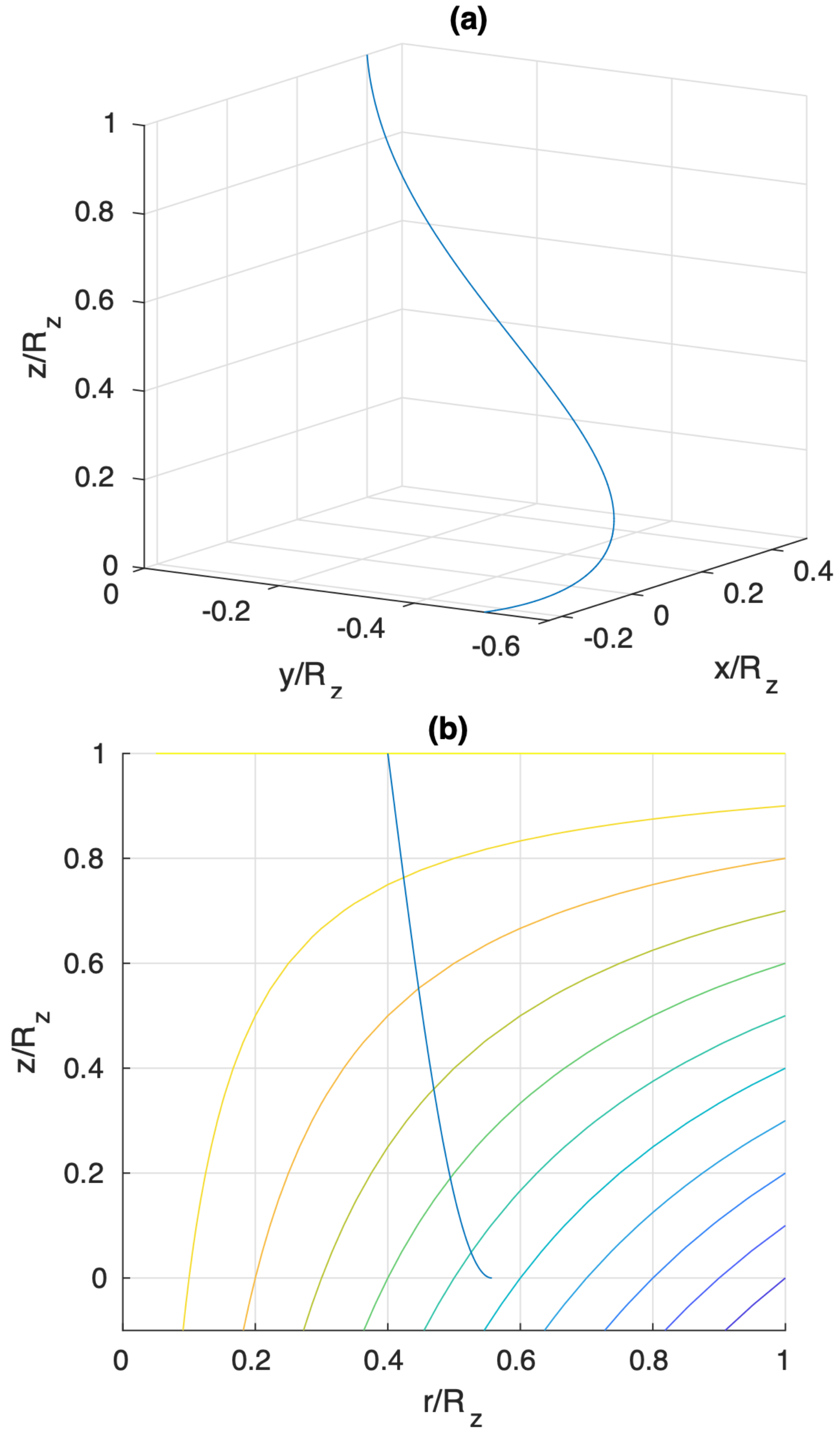
The expression of the reference orbit is given by solving the motion equation

$$r = R_r b + r_0$$

$$\theta = \int_0^b \frac{\sin(b)}{\frac{R_r}{R_z} b + \frac{r_0}{R_z}} db$$

$$z = R_z sin(b) + z_0$$

Where $R_r = p_{r0}/qB_0$ and $R_z = pz_0/qB_0$, The momenta in z direction arrives at 0 at the median plane where $b=\pi/2$.



Magnetic mirror

The magnetic mirror is a component used to confine the charged particles. A vector potential used to define the axial symmetric magnetic field in a magnetic mirror is given as below

$$A_{theta} = \frac{A_1 \beta r}{2} - A_2 I_1(\beta r) \cos \beta z$$

The given magnetic field satisfies the Laplace's equation, which ensures that the field could be designed in a free space. The linear approximation of the vector potential is given as

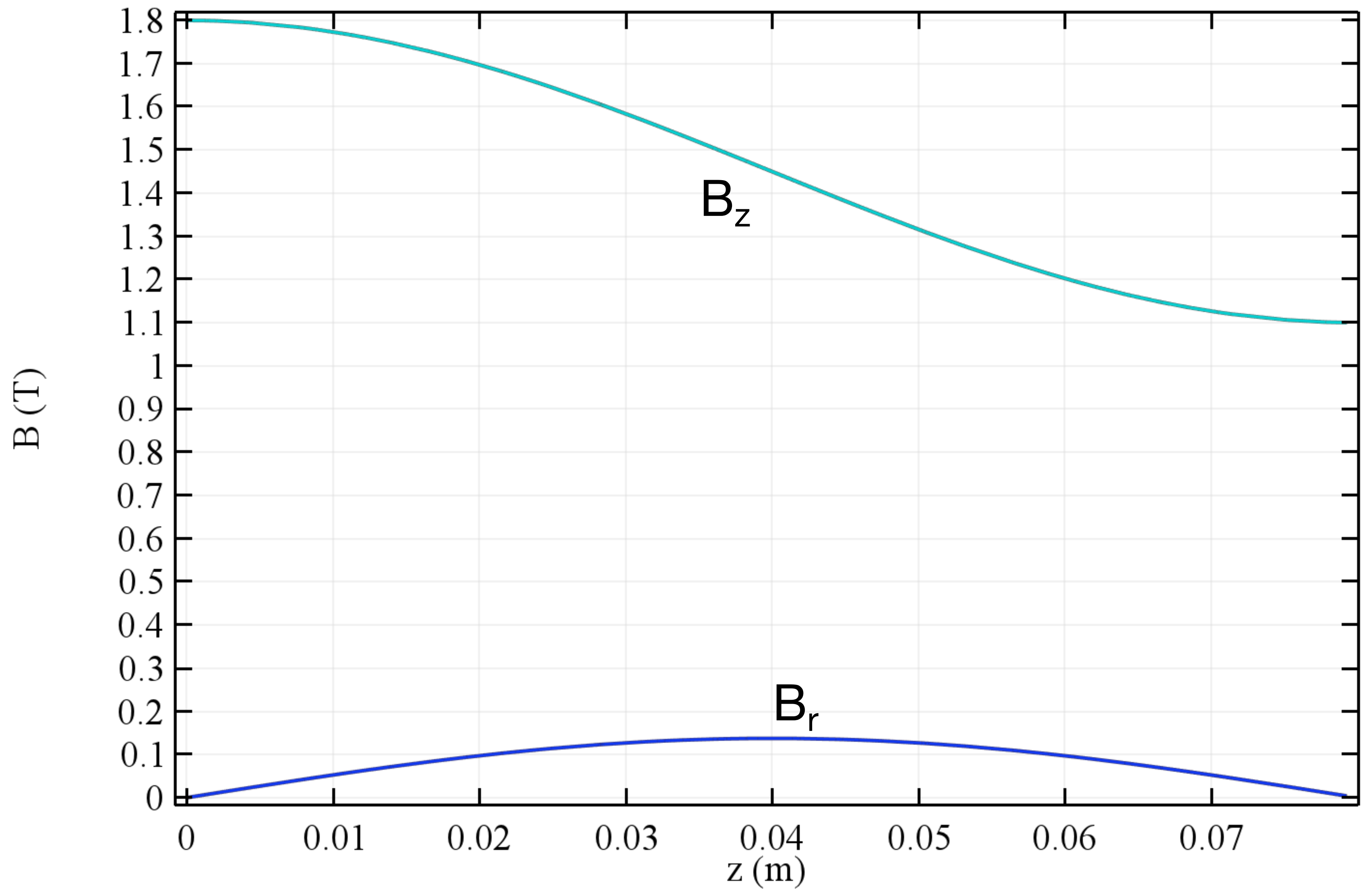
$$A_{theta} = \frac{\beta r}{2}(A_1 - A_2 \cos \beta z)$$

Magnetic field is written as

$$B_r = -(\beta^2 r A_2/2) sin(\beta z)$$
$$B_z = \beta(A_1 - A_2 cos(\beta z))$$

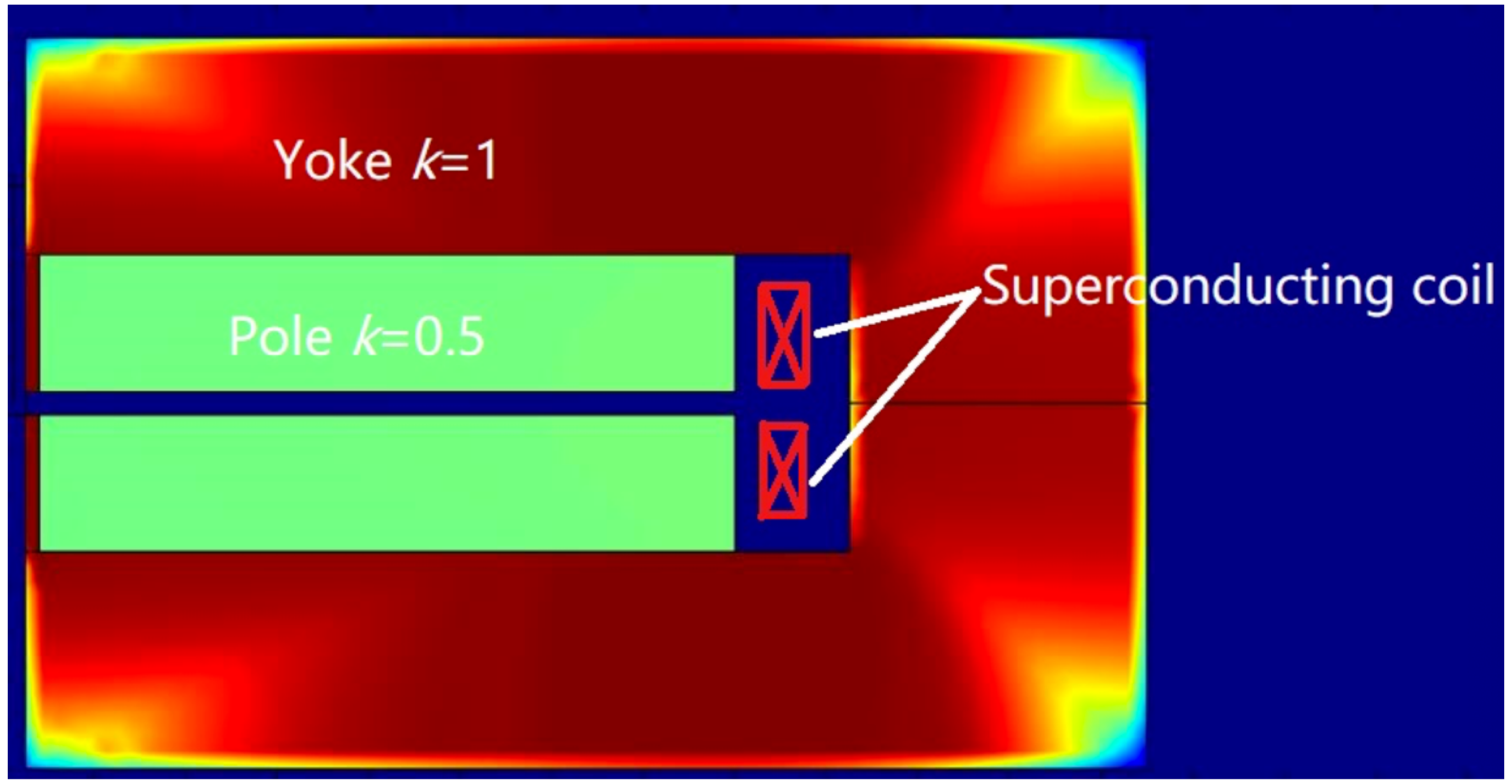
The motion equation could be written as a 2D form in r-z plane

$$P'_r = \frac{\partial U}{\partial r}$$
$$P'_\theta = 0$$
$$P'_z = \frac{\partial U}{\partial z}$$

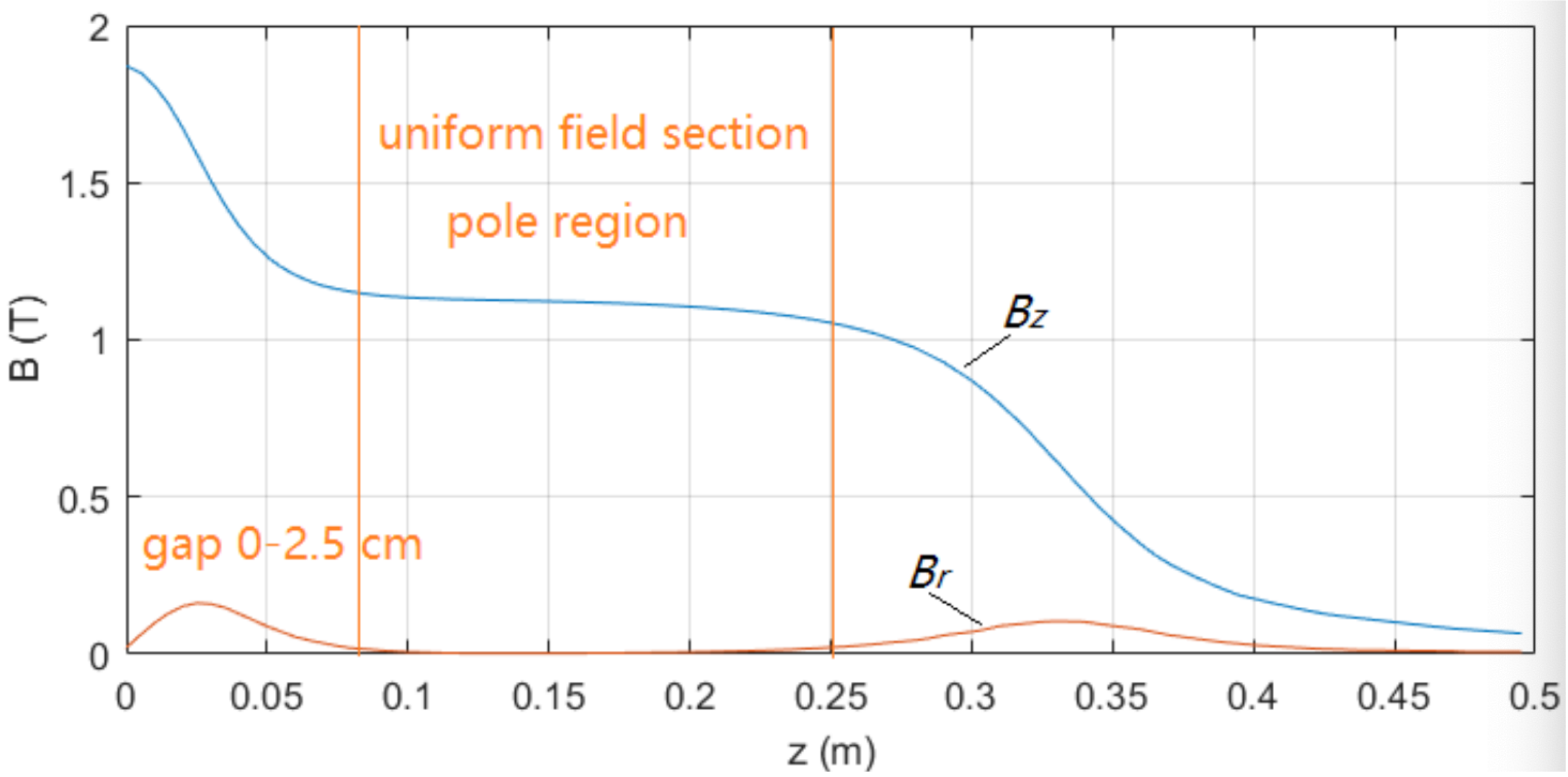


Magnetic field in the injection hole

Parameters	Value
Particle accelerated	H_2^+
Gap (cm)	5
Number of sectors	4
B0 at centre (T)	2.0
Pole radius (m)	1.65
Injection scheme	Axial + external ion source
Coils	2 superconducting coils

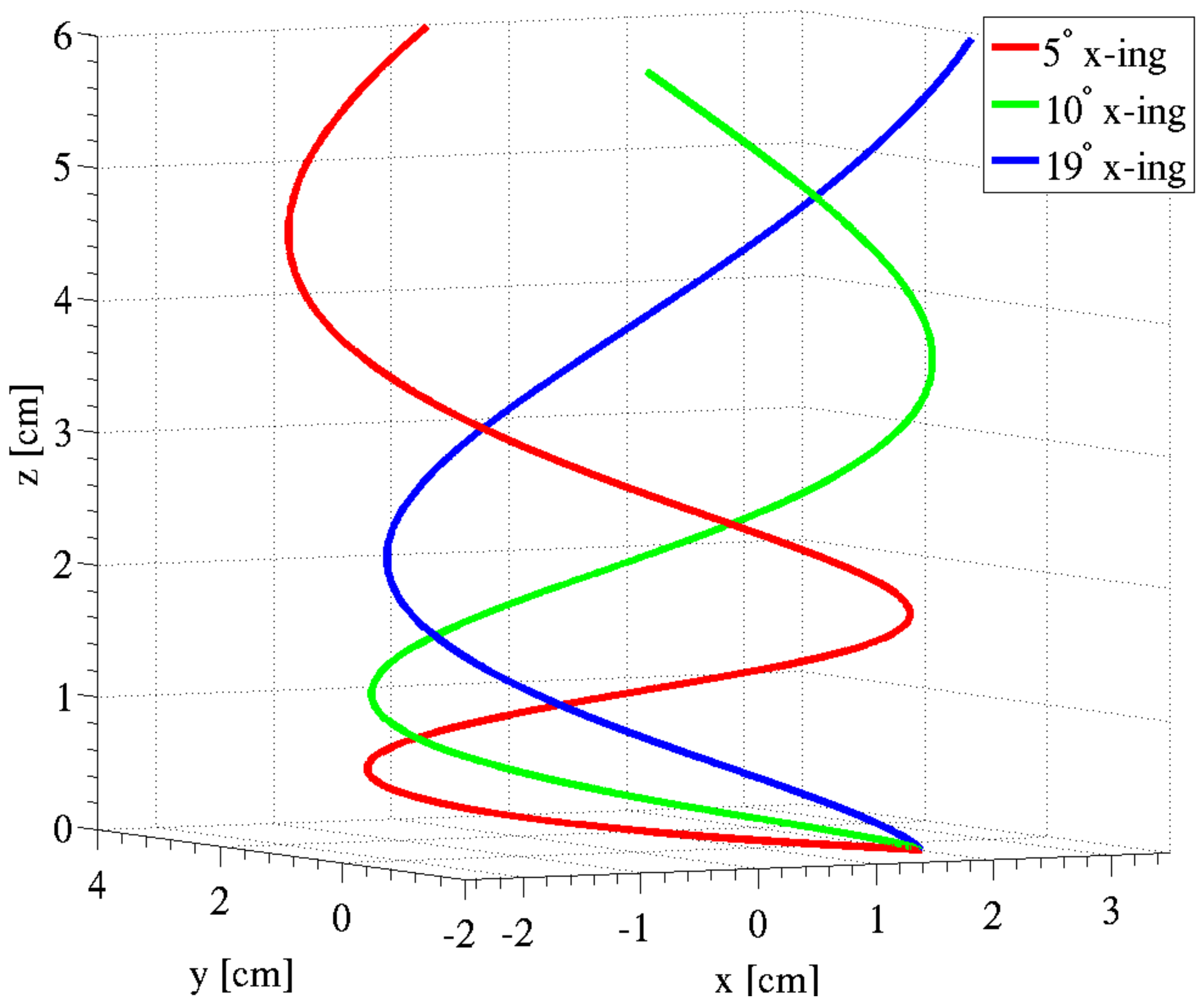


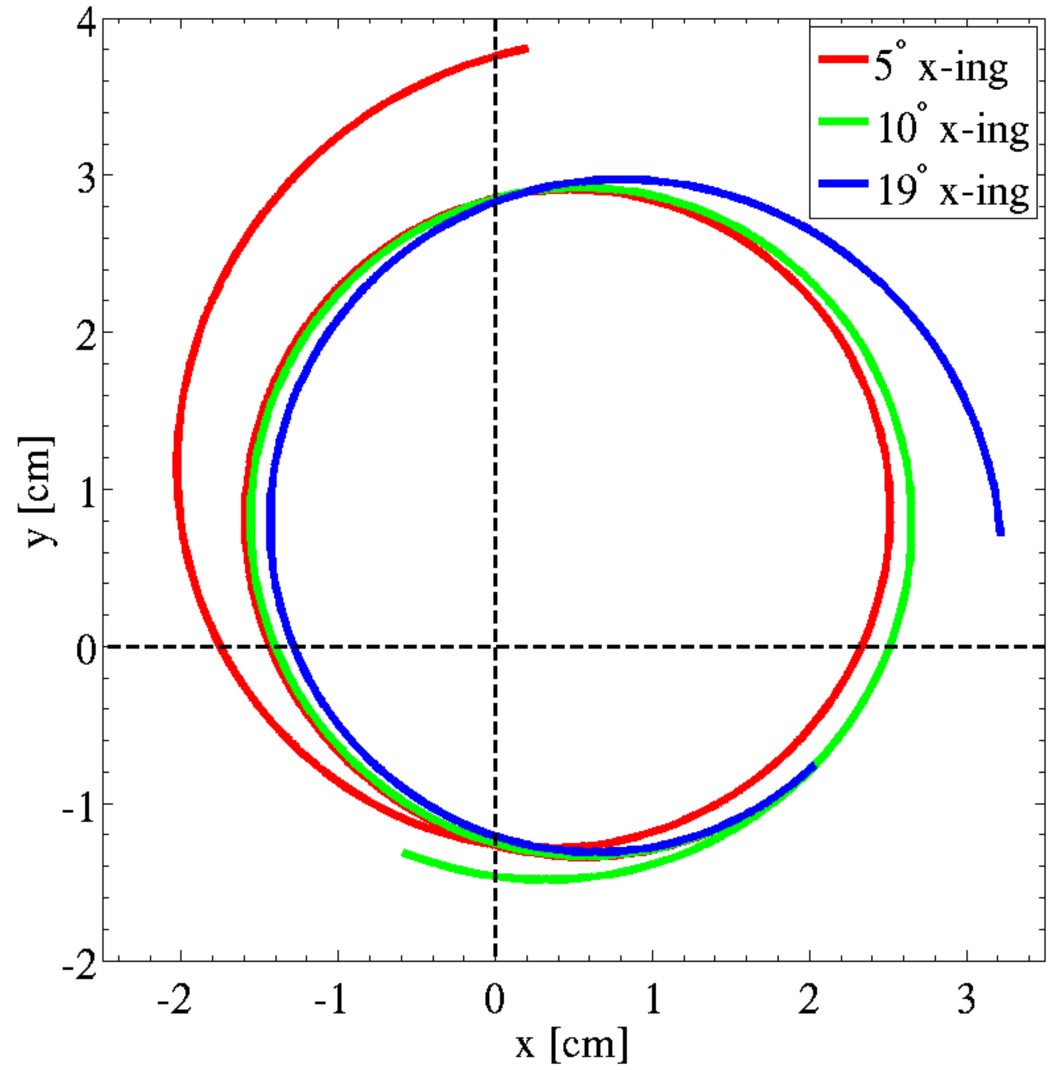
Magnetic field along r=1 cm. The field could be recognised as 3 sections which reflect the gap, pole and yoke structure of the magnet.



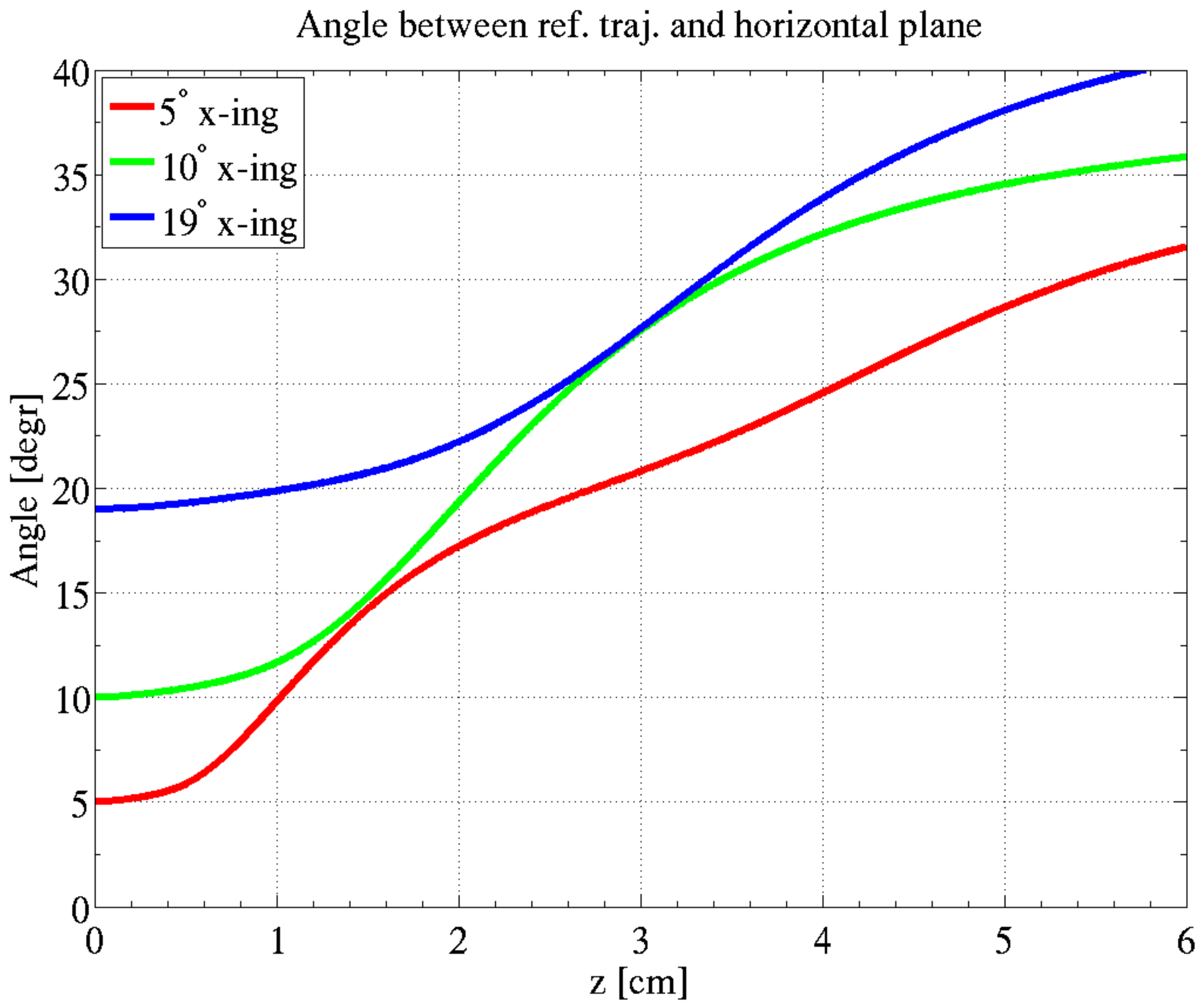
Reference orbit

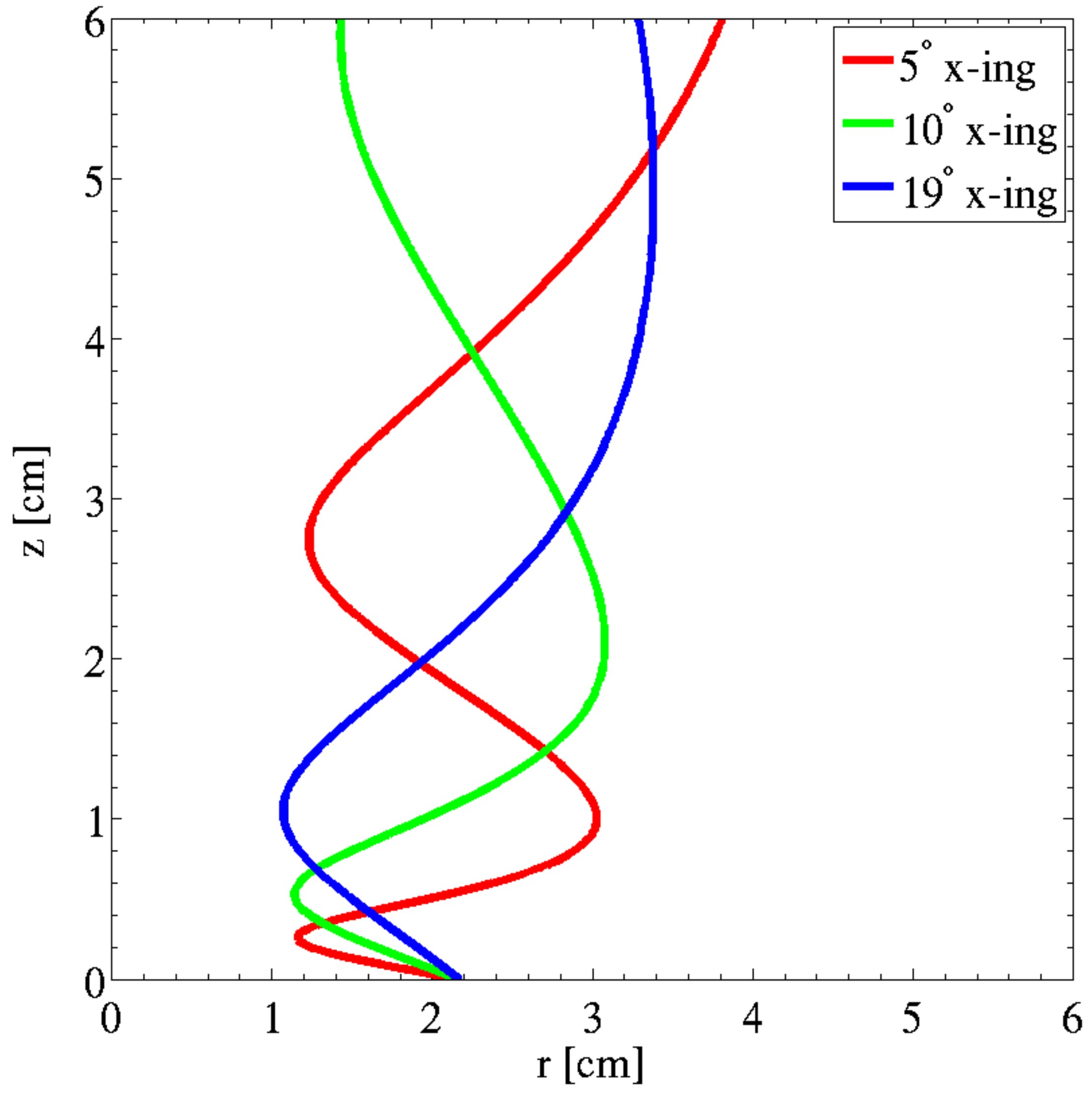
Track the particle reversely from the median plane to the injection point with different Pitch angles





The single B_r bump field near the median plane could reduce the pitch angle by about 20° from the injection point to the median plane



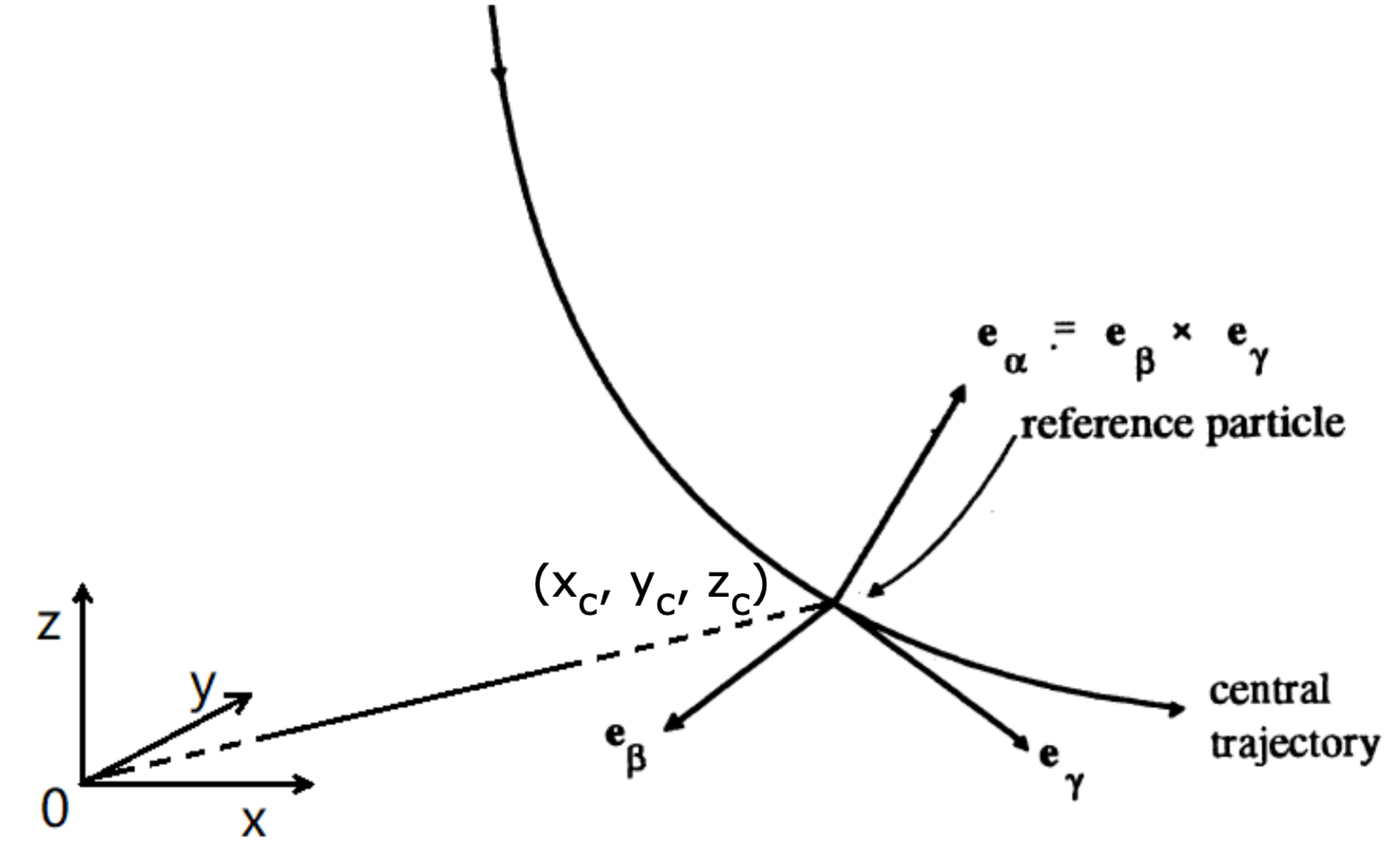


Moving frame

The moving frame $\alpha\beta\gamma$ is a right handed coordinate system. The γ direction is the same with the velocity of the reference particle. The β direction is perpendicular to the γ direction and parallel to the median plane.

The transformation from (x,P_x,y,P_y,z,P_z) to $(\alpha,P_\alpha,\beta,P_\beta,\gamma,P_\gamma)$ is given by

$$\begin{bmatrix} \alpha \\ P_\alpha \\ \beta \\ P_\beta \\ \gamma \\ P_\gamma \end{bmatrix} = \mathrm{M}^T \begin{bmatrix} x - x_c \\ m_0 v_0 (x' - x_c') \\ y - y_c \\ m_0 v_0 (y' - y_c') \\ z - z_c \\ m_0 v_0 (z' - z_c') \end{bmatrix} + q\mathrm{M}^T \begin{bmatrix} 0 \\ A_x - A_{x0} \\ 0 \\ A_y - A_{y0} \\ 0 \\ A_z - A_{z0} \end{bmatrix}$$



Transfer matrix

Calculate the transfer matrix by tracking 6 particles numerically with different initial coordinates and momenta

$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (1,0,0,0,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,1,0,0,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,1,0,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,0,1,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,0,0,1,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,0,0,0,1)$$

Transfer matrix in the moving frame

$$\mathrm{R} = \begin{bmatrix} 1.9899 & 0.1493 & -1.6822 & -0.0167 & 0.3753 & 0.1340 \\ -5.0231 & 0.1862 & -0.2335 & -0.1780 & 3.8668 & 0.2139 \\ 0.5800 & 0.0223 & 0.8386 & 0.0260 & -0.5524 & -0.0134 \\ -13.7973 & -0.3713 & -8.0835 & 0.3925 & 1.9409 & -0.6353 \\ -0.0311 & 0.0394 & -0.3041 & 0.0195 & 0.6095 & 0.0989 \\ 5.3240 & 0.2201 & -12.4786 & 0.2672 & -5.4282 & 0.8583 \end{bmatrix}$$

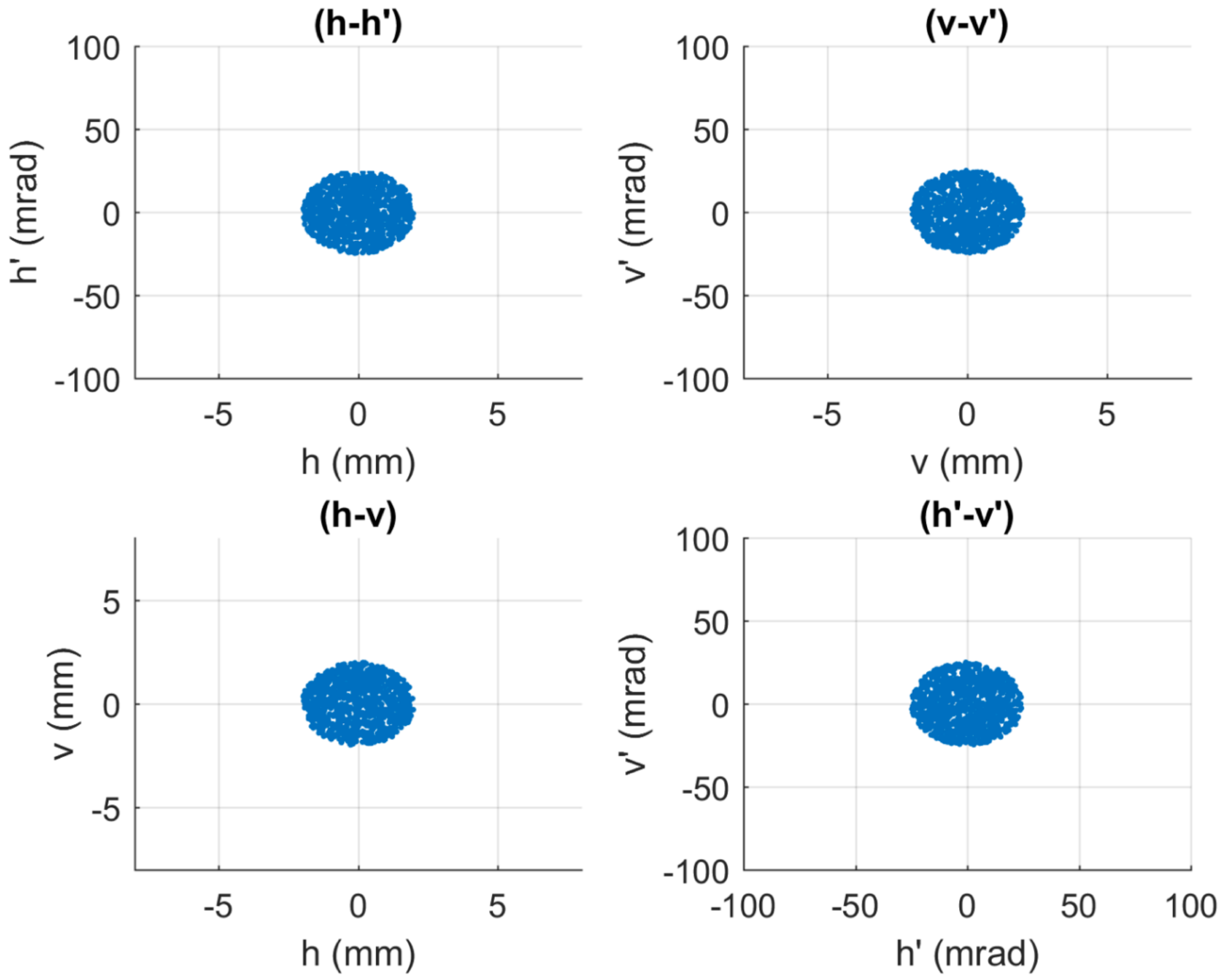
Transfer matrix symplectic

$$\mathrm{R^TJR - J} = \begin{bmatrix} 0 & -0.0043 & -0.0258 & 0.0364 & 0.0082 & -0.0084 \\ 0.0043 & 0 & -0.0151 & 0.0012 & -0.0026 & -0.0001 \\ 0.0258 & 0.0151 & 0 & -0.0024 & 0.0014 & 0.0036 \\ -0.0364 & -0.0012 & 0.0024 & 0 & 0.0007 & -0.0006 \\ -0.0082 & 0.0026 & -0.0014 & -0.0007 & -0.0000 & -0.0008 \\ 0.0084 & 0.0001 & -0.0036 & 0.0006 & 0.0008 & 0 \end{bmatrix}$$

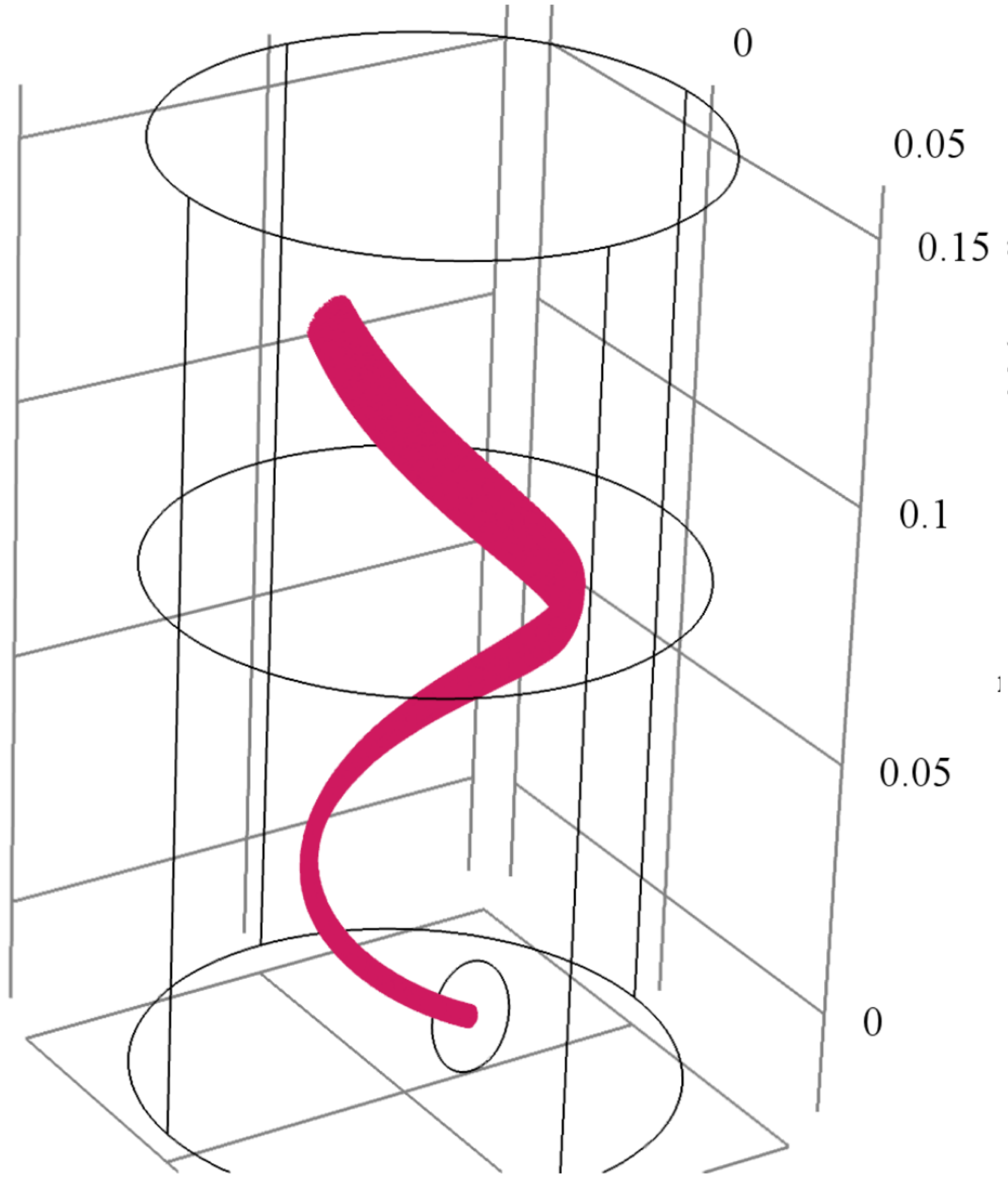
$$\mathrm{J} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The error is between 10^{-3} and 10^{-2}, which may be resulted from the non-linear of the motion and the noise when tracking the particles numerically.

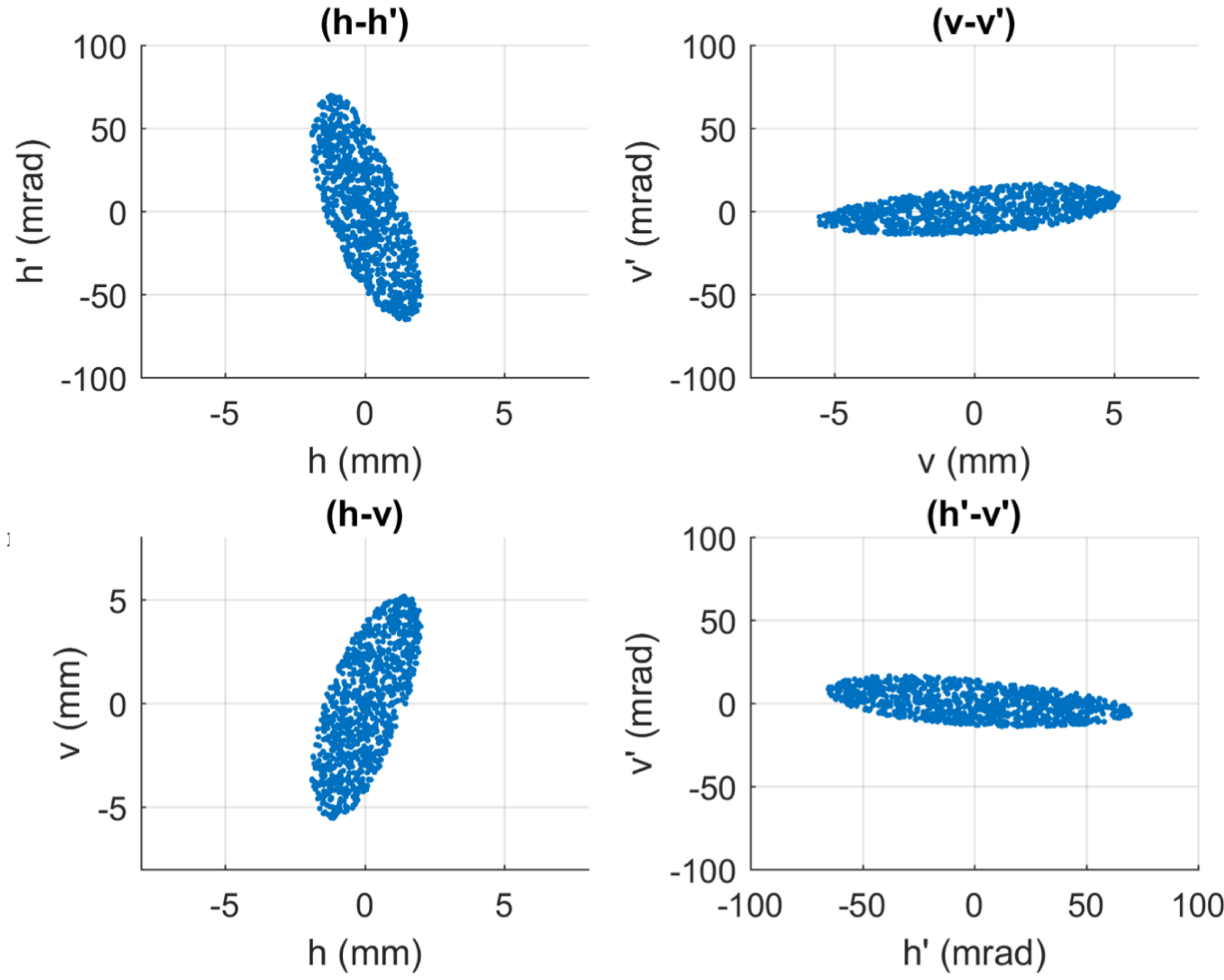
Beam focusing



Initial K-V distribution beam at the matching point on the median plane



Particle orbits in cartesan frame



Phase plot at the injection point

Discussion

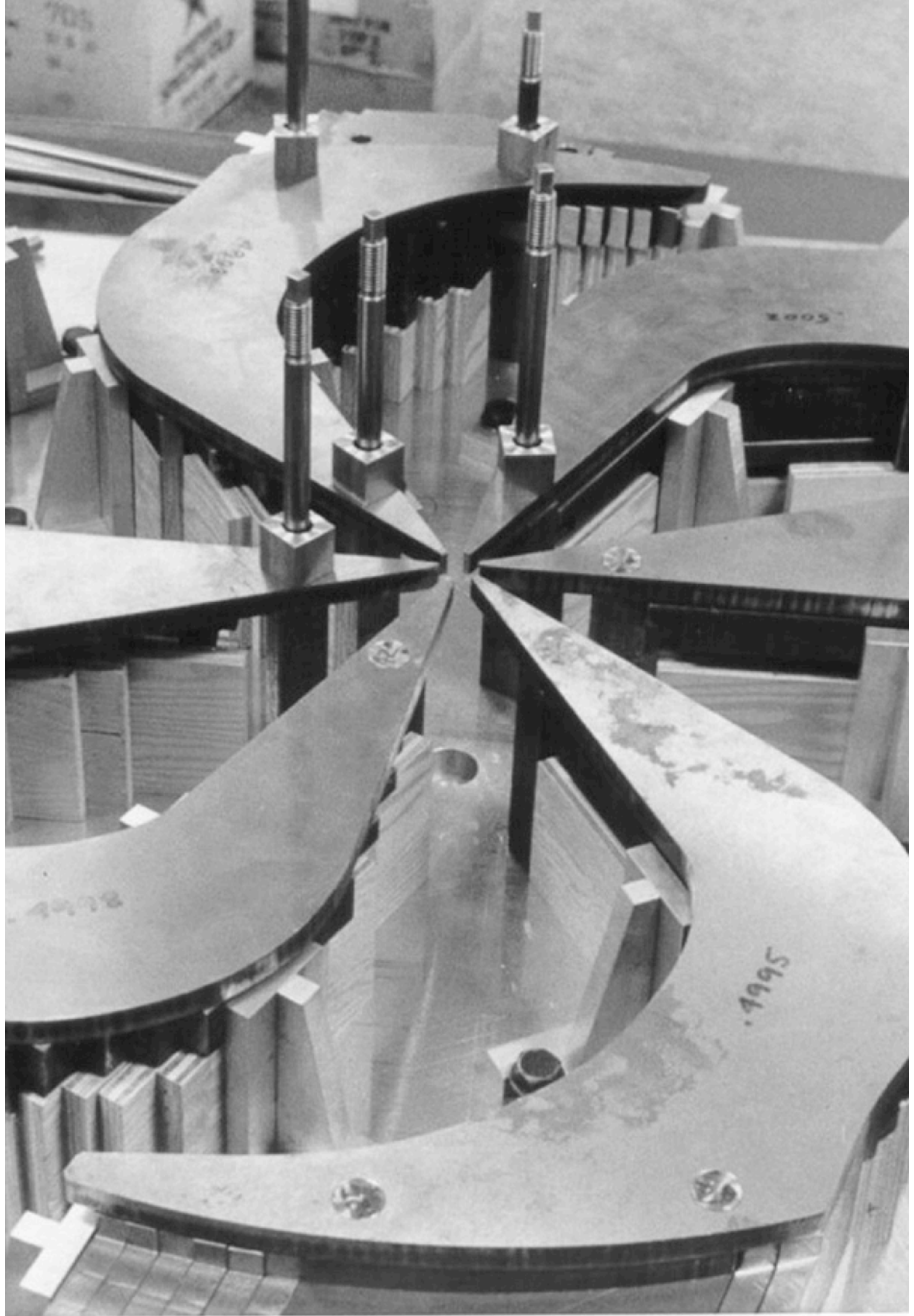
- Optimize the reference orbit to achieve larger pitch angle at the injection point
- To keep the median plane symmetry, a electrostatic plate should be used at the end of magnetic inflector, which will finally deflect the beam vertically into the median plane
- Method to focus the beam in the vertical direction

TRIUMF

Thank you
Merci

www.triumf.ca

Follow us **@TRIUMFLab**



Discovery, accelerated