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Orbit Correction using FFA Correctors in CBETA

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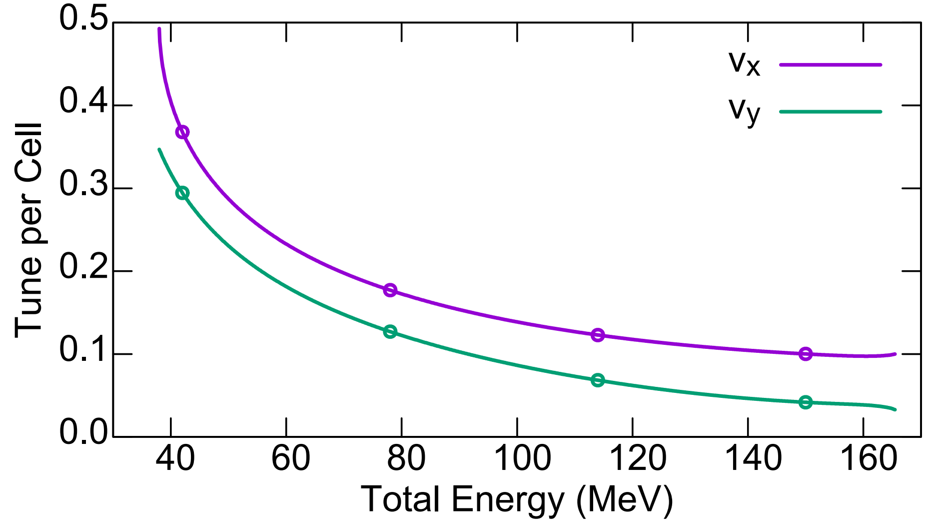
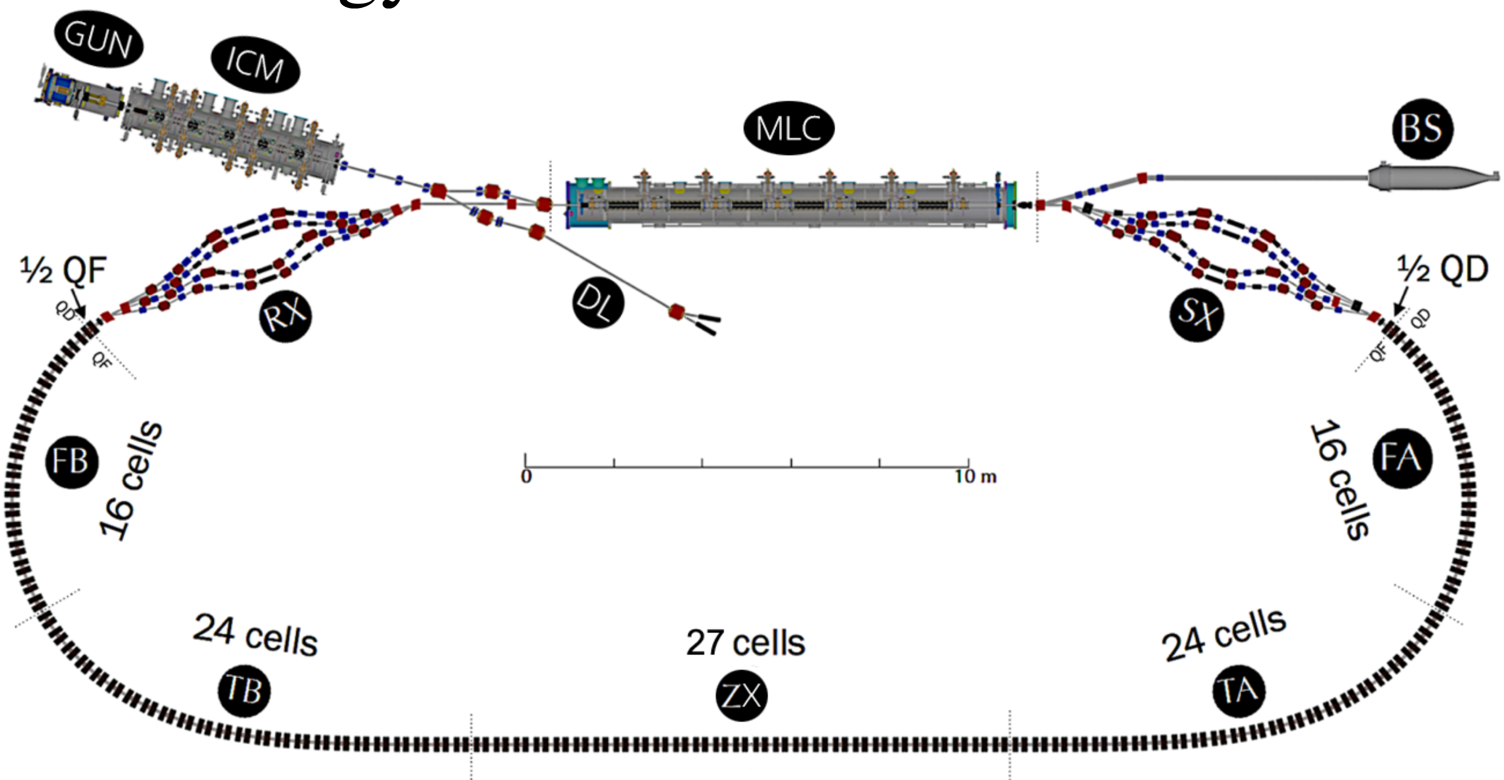
Overview

- Talking only about orbit correction using FFA correctors
  - Beams with multiple energies passing through correctors
  - Only four energies (not 100s or 1000s of turns)
  - Different phase advances mean correctors act independently on different energies
- In some cases correcting orbit outside of FFA (RX lines)
- Work of A. Bartnik, S. Brooks, K. Deitrick, C. Gulliford, and myself



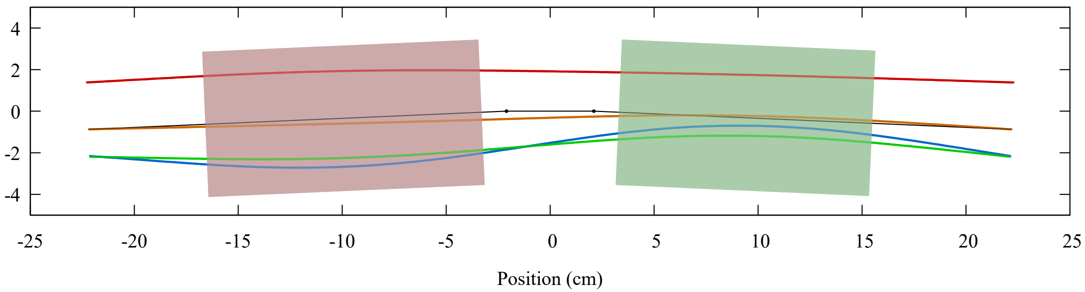
CBETA Configuration

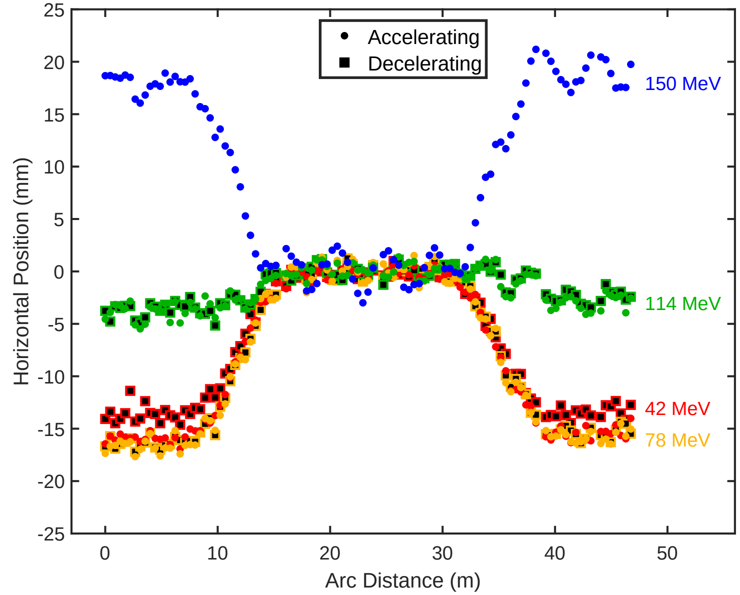
- FFA contains arcs (FA/FB), a straight (ZX), and transitions (TA/TB)
- Connected to a linac by splitter lines (SX/RX), one for each energy



CBETA Configuration

- FFA cell is a doublet
- Every focusing magnet has a horizontal corrector, every defocusing magnet has a vertical corrector
- One BPM per FFA cell, same location in each cell
  - Can get beam position for each pass
- Four energies in FFA: 42, 78, 114, and 150 MeV





Orbit Correction Methods

- Two methods used in CBETA operation
  - Straightforward SVD correction of entire orbit
  - Fixed subspace correction, allowing correction of part of one pass while leaving earlier passes unchanged

SVD Orbit Correction

- Least squares minimize error in BPM readings y_{ij} on pass i on BPM j
- Linear response of y_{ij} to corrector c_k

$$\frac{\partial y_{ij}}{\partial c_k} = R_{ij;k}$$

- Singular value decomposition (SVD): write

$$R = U\Sigma V^T = \begin{bmatrix} U_A & U_B \end{bmatrix} \begin{bmatrix} \Sigma_A \\ 0 \end{bmatrix} V^T$$

  - Σ_A is diagonal, entries are positive, and ordered from largest to smallest
  - U and V are orthogonal

SVD Orbit Correction

- In principle $\Delta \boldsymbol{c} = -V\Sigma_A^{-1}U_A^T\boldsymbol{y}$
  - Finds the correction which
    - Minimizes the sum of the squares of the orbit errors, or
    - Minimizes the sum of the squares of the corrections
  - But measurement errors can lead to large corrections due to small entries in Σ_A
  - Typically use only large values in Σ_A

$$R = U\Sigma V^T = \begin{bmatrix} U_{A1} & U_{A2} & U_B \end{bmatrix} \begin{bmatrix} \Sigma_{A1} & 0 \\ 0 & \Sigma_{A2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

    - Now $\Delta \boldsymbol{c} = -V_1\Sigma_{A1}^{-1}U_{A1}^T\boldsymbol{y}$
    - Not done in operation; targeted corrector strengths (below)
- Real problem is nonlinear; iterate to convergence

Corrector Strengths

- Prevent correctors from running out of strength
  - Append corrector strengths (not the changes in the corrector strengths) to vector of orbit positions being corrected
  - Relate corrector strengths to orbit errors with empirically determined weight factor
- Multiply corrector changes by a factor
  - Model/measurements imperfect, prevent unreasonable corrections
  - Multiply computed changes by a factor $\kappa < 1$
  - κ varied from 0.1 to 1, depending on region we worked in, etc.

Horizontal Orbit Position

- Horizontal target orbit position is unknown
  - Model and BPM systematics
  - Should be periodic in FA/FB arcs
  - Known to be zero in straight ZX
  - In TA/TB transitions, change from zero to arc value
- Adjust corrector strengths $\boldsymbol{c}$ to correct

$$x_{ij}(\boldsymbol{c}) - \lambda_{ij}\hat{x}_i$$

solving for arc orbit $\hat{x}_i$ (pass i) in the process

  - $\lambda_{ij} = 1$ in the arcs, 0 in the straights, $0 < \lambda_{ij} < 1$ in the transitions, based on the model

Horizontal Orbit Position

- Two methods:
  1. Add columns to the SVD for $\hat{x}_i$
     - What we used in practice
     - Has some mathematical issues
  2. Use the analytic least squares value for $\hat{x}_i$

$$\hat{x}_i = \sum_j \lambda_{ij} x_{ij} \Bigg/ \sum_j \lambda_{ij}^2$$

     - Treat this as exact. Effective response matrix

$$R_{ij;k} - \lambda_{ij} \sum_l \lambda_{il} R_{il;k} \Bigg/ \sum_l \lambda_{il}^2$$

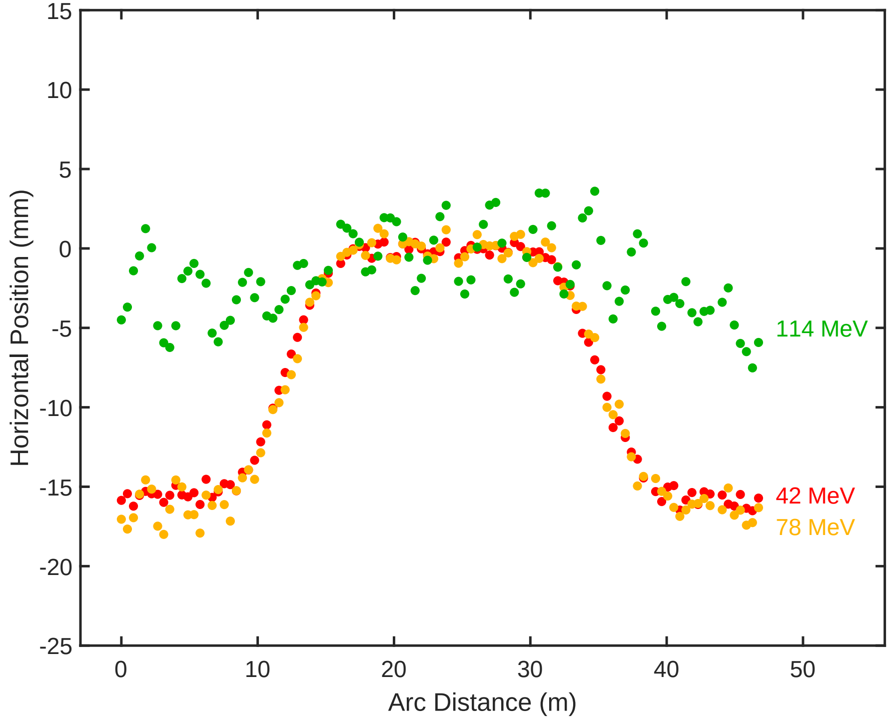
Response Matrix

- Would like to use measured response matrix, but:
  - Measurement is time-consuming
  - Issues in turn-to-turn response
  - Beam loss on later turns
- Methods used
  - Model response matrix
    - Only used for very short ranges
    - Chromaticity combined with systematic energy/model error gives problems for long range
      - E.g., 300 kV energy error, response sign wrong at end
  - Model assuming constant phase advance
    - Measure cell phase advance
    - Corrector-to-bpm response from model
    - Each section (FA/TA/ZX/TB/FB) done separately

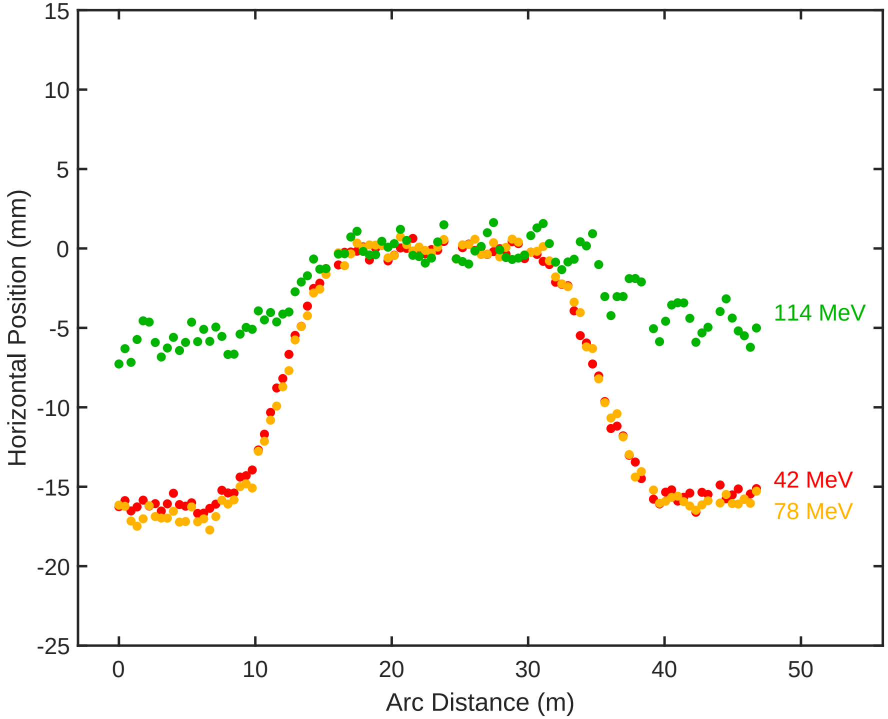
Decoupling and Result

- Decouple turns when correcting
  - Heavily weight end of FB to leave positions unchanged
- Result for three turns
  - Improved, but not perfect

Before Correction



After Correction



Fixed Subspace Correction

- SVD correction did the bulk of our correction, but wanted some cleaning up, particularly on later turns
- Want a method that can
  - Make corrections in a local region on later turns, while leaving earlier turns unchanged
  - Measure response matrices, without losing beam on later turns
  - Problem: all turns pass through the same correctors
- Fixed subspace correction
  - Correct the orbit for a subset of BPMs on a given turn
  - Using combinations of correctors that leave the orbit on those BPMs unchanged for earlier turns

Fixed Subspace Correction

- Problem setup
  - Have n correctors c_i, $1 \leqslant i \leqslant n$
  - Read m BPMs downstream of all of these correctors, reading positions $y_{i;j}$ for BPM j on pass i, $1 < i < p$, $1 < j < m$
  - Correct the orbit on the BPMs for pass p, leaving the orbit unchanged on the BPMs for passes 1 through $p - 1$
  - Can instead correct orbit at BPMs in RX lines.
    - “Pass” becomes replaced with index of RX line.
    - Correcting vertical orbit in the RX lines using late FB correctors was the initial motivation and the most important application of the technique.

Fixed Subspace Correction

- Start by measuring the response matrix for pass 1

$$\frac{\partial y_{1;i}}{\partial c_j} = R_{1;ij}$$

  - Orbit downstream of the correctors is defined by two quantities (position and angle)
  - Thus R_1 should be rank 2
    - There are only 2 independent combinations of BPM readings that are possible
    - But more than 2 independent combinations of correctors can produce those combinations of BPM readings

Fixed Subspace Correction

- Due to measurement noise and numerics, the matrix will not be perfectly rank 2
  - Need a method to construct a rank 2 matrix
  - Easiest is to use the SVD. Block form

$$R_1 = \begin{bmatrix} U_{11} & U_{12} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{12} \end{bmatrix} \begin{bmatrix} V_{11}^T \\ V_{12}^T \end{bmatrix}$$

  - Σ_{11} is the 2×2 block with the largest singular values
- Corrector strengths are given by

$$\boldsymbol{c}_1 = \boldsymbol{c}_0 - V_{11}\Sigma_{11}^{-1}U_{11}^T\boldsymbol{y}_1$$

  - Combination is not unique. SVD chooses the combination that minimizes the sum of the squares of the corrector changes.
- Corrector combinations in the range of V_{12} should leave pass 1 unchanged

Fixed Subspace Correction

- For pass 2, use combinations that leave the pass 1 orbit through the BPMs unchanged
  - Use $n-2$ corrector combinations $V_{12}\bar{\boldsymbol{c}}_2$
    - $\bar{\boldsymbol{c}}_2$ is $n-2$ dimensional, V_{12} is $n \times (n-2)$
  - Measure response matrix of pass 2 BPM readings to $\bar{\boldsymbol{c}}_2$

$$\frac{\partial y_{2;i}}{\partial \bar{c}_{2;j}} = R_{2;ij}$$

Control system varies elements of $\bar{\boldsymbol{c}}_2$, not single correctors! Prevents beam loss since pass 1 (nearly) unchanged.

Fixed Subspace Correction

- As before, construct a block SVD of this response matrix

$$R_2 = \begin{bmatrix} U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} \Sigma_{21} & 0 \\ 0 & \Sigma_{22} \end{bmatrix} \begin{bmatrix} V_{21}^T \\ V_{22}^T \end{bmatrix}$$

  Again, Σ_{21} is a 2×2 block with the largest singular values

- Using only the rank 2 part to invert,

$$\boldsymbol{c}_2 = \boldsymbol{c}_1 - V_{12} V_{21} \Sigma_{21}^{-1} U_{21}^T \boldsymbol{y}_2$$

- Can be extended to any pass p

$$\boldsymbol{c}_p = \boldsymbol{c}_{p-1} + V_{12} \cdots V_{p-1,2} \bar{\boldsymbol{c}}_p$$

etc.

Dispersion

- Can correct dispersion instead of orbit, since dispersion wave downstream of correctors defined by two quantities
- Can we correct both dispersion and orbit? Sort of.
- Procedure as before, except
  - Measure both dispersion and orbit
  - Subspace used for inversion is 4-dimensional instead of 2-dimensional
- Two difficulties
  1. Dispersion and orbit are different scales and dimensions
     - Address with scaling factors

Dispersion

2. Correctors affect orbit and dispersion in the same way

- Equations for orbit and dispersion corrector response

$$(\Delta x)'' + K_x(\Delta x) = -\Delta h \quad (\Delta D)'' + K_x(\Delta D) = -(\Delta h)hD$$

  h and D are ideally periodic, so the D response is proportional to the x response
- Errors break the symmetry. Also different to nonlinear order in the corrector strength.
  - Thus, in some cases we can correct simultaneously
  - But this leads to a large corrector strengths and orbit excursion in the corrector region
  - In practice examined magnitude of 3rd and 4th singular values to decide whether simultaneous correction made sense
- Better way to correct orbit and dispersion: quadrupole correctors

$$(\Delta D)'' + K_x(\Delta D) = -(\Delta h)hD - (\Delta K)D$$

Summary

- Our initial FFA orbit correction was performed with a straightforward SVD correction
  - We used a model assuming constant phase advance per cell to get a response matrix
  - Turns were decoupled by preventing orbit changes at the ends of the FFA arc
  - We solved for the unknown periodic orbit in the FFA arc sections and the tapered orbit in the transition region
- Orbit cleaned up with fixed subspace correction
  - Corrections are found leaving earlier orbits unchanged
  - RX orbits (mainly vertical) can also be corrected
  - Dispersion can be corrected instead
    - Simultaneous correction of orbit and dispersion presents difficulties