An analytic approach to modelling the VFFA

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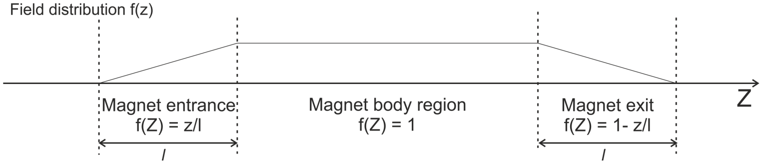
Motivation – why do we need to develop new theory for the VFFA?

- VFFA is an interesting candidate machine for a number of applications:
  - Neutron spallation sources
  - Muon acceleration
- However, scaling criterion $B(Y) = e^{mY}$ implies highly coupled + nonlinear dynamics
  - Including potentially strong solenoid components in the fringe fields
- Hence, current models require a full numerical simulation of the machine
  - Computationally intensive and time-consuming
  - Simulations are lattice-specific
  - Limited understanding of how input parameters affect output parameters
- An analytic approach could provide a cheap way of developing and optimising new lattices!

A Simplified Hamiltonian Approach



- Break magnet down into a 3-element approximation
  - Short fringe 'entrance' region
  - Long magnet body
  - Short fringe 'exit' region
- Construct a vector potential for each region and insert into the accelerator Hamiltonian
- Expand to linear order and apply Hamilton's equations

Hamiltonian analysis – magnet body

VFFA Magnet Body Hamiltonian

$$\mathcal{H} \simeq \frac{1}{2}P_x^2 + \frac{1}{2}P_y^2 + \frac{1}{\rho}x + \frac{1}{\rho^2}x^2 + \frac{1}{\rho}mxy, \tag{1}$$

x, P_x - horizontal position, canonical momentum; y, P_y - vertical position, canonical momentum ρ - radius of curvature

Can be broken down into:

- A dipole bending term
- A weak focusing term
- A skew quad-like term

Hamiltonian analysis – magnet fringe

VFFA Magnet Entrance Hamiltonian*

$$\mathcal{H} \simeq \frac{1}{2}p_x^2 + \frac{1}{2}p_y^2 + \frac{s}{l}\frac{1}{\rho_b}x + \frac{s}{l}\frac{1}{\rho_b^2}x^2 + \frac{s}{l}\frac{1}{\rho_b}mxy - \frac{1}{2}\frac{1}{ml\rho_b}p_y x - \frac{1}{2}\frac{1}{ml\rho_b}p_x y, \quad (2)$$

ρ_b - radius of curvature in magnet body, l - length of fringe field region

Can be broken down into:

- An s-dependent dipole bending term
- An s-dependent weak focusing term
- An s-dependent skew quad-like term
- And a constant solenoid-like term

*The equation displayed is written in terms of kinetic, rather than canonical, momentum, in order to make the solenoid component more apparent

Testing the model

- Integrating these equations of motion is nontrivial!
- Having a way of checking that our ‘building blocks’ for the analytic description are sending us in the right direction
  - i.e. can we check the validity of the model before working each part of it through to completion?

Using existing tools to model the VFFA

- Can we use MADX for a first order approximation?
  - Widely-used and familiar!
  - Lots of basic elements we can use
  - Can define our own elements if we know the transfer matrix
  - Access to Ripken decoupling parametrisatoin using PTC interface
- However, user defined MATRIX entity not supported by PTC
  - MADX's native handling of coupled dynamics is not good
- Can we use out-of-the-box elements to build a rough picture of the VFFA?
  - Skew quad body + solenoid end fields
  - MADX solenoids include solenoid fringe, solenoid body, solenoid fringe as a package – we cannot separate the effect we are interested in (and hence we gain unwanted extra focusing)

Developing a new tool

- Specification:
  - Transfer matrix-based
  - MADX-like
  - Python module
  - Native implementation of decoupling parametrisations

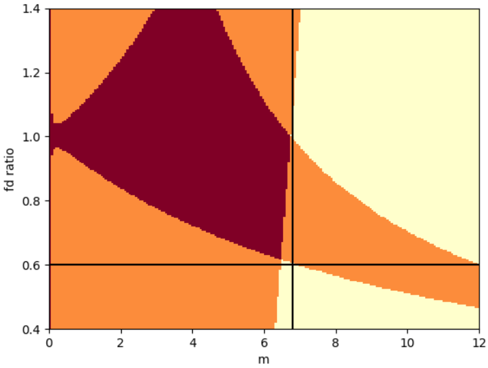
    ...
  - and include VFFA elements as standard!

Testing the linearised model of the VFFA

- My approximations:
  - Paraxial approximation
  - Small bending angle per cell
  - No edge focusing
  - 3 distinct elements contribute to whole magnet
- Test lattice
  - Muon VFFA Lattice desgned by S. Machida, presented in August 2020
  - 810 cell FODO, 50GeV to 1.5TeV muon accelerator ring
  - FD ratio (ratio of strength of reverse bend magnet to normal bend magnet) as free variable

	FODO
Energy	50 GeV to 1.5 TeV
Cell length	35 m
Magnet length	2 x 15 m
# of cell	810
Maximum field	8.7 T
Field index m	6.8
Orbit excursion	0.50 m
Cell tune	0.3957 / 0.0861

Parameter scan results

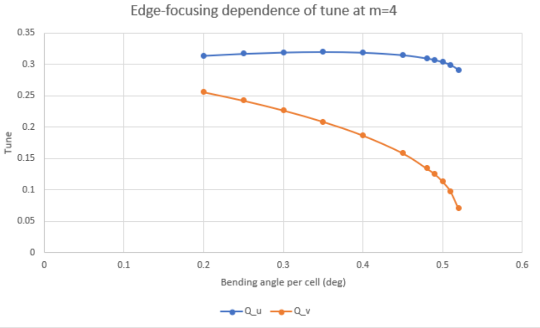


- Orange shows where one mode is stable; Red where two modes are stable; light yellow where there is no stable mode
- Black lines indicate the parameters set by S. Machida
- Clearly, my model predicts a different stability region

Interpretation

- The agreement between my model and the simulations of S. Machida is very low under the approximations made
- For the muon collider lattice we should have:
  - Small bending angle per cell
  - Short fringe field compared to magnet body
  - Large radius of curvature
- All of the above should be okay for this linearised approximation
- Further testing using the model of S. Machida shows strong effects of edge focusing

Effects of edge focusing



- Even for small edge angles the effect is significant
- Clearly, it is necessary to consider edge focusing to accurately model the VFFA machine!

Conclusions and further development

- We have a new, transfer matrix-based optics code specifically tailored to the VFFA
  - Native access to decoupling parametrisations
- However, current formulation of linearised model under my original set of approximations is insufficient for modelling the VFFA
- Clearly, it is necessary to consider edge focusing to accurately model the VFFA machine!